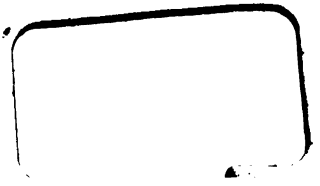
KEY
TO
TOWNE'S ALGEBRA



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KEY

TO

TOWNE'S ALGEBRA.

PREPARED BY

THE AUTHOR.

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KEY TO TOWNE'S ALGEBRA.

In this Key, the heavy figures on the left of each page refer to the *page* of the Algebra, and the light figures to the problems on the page.

EXAMPLES

INVOLVING THE PRECEDING DEFINITIONS.

14. 3. $(x+y)^{\frac{1}{5}}$ or $\sqrt[5]{x+y}$.

6. (1.) Twelve times the product of x and y, increased by the square root of seven times the third power of x, taken from five times the fourth power of x.

6. (2.) The square root of *the sum of the square of the cube root of* x, *and the reciprocal of the fifth root of the cube of* x, increased by x.

6. (3.) The square root of *the sum of the square of* x, *and the square root of the fifth taken from the sum of the third and fourth powers of* x, increased by x.

15. 7. (1.) The square root of *the difference found by taking the square root of the sum of five times the cube of* x *and three times* x *from four times the square of* x, increased by the product of x and y.

7. (2.) The square of x divided by the cube of y equals seven times the square of x increased by eight times the product of a and m.

7. (3.) The sum of x and y divided by their difference is greater than the cube root of the difference between twice a and five times b.

8. (1.) The quotient of c divided by two is less than the quotient

arising from dividing *the square of b taken from the sum of the square of a and the square of c by twice c.*

15. 8. (2.) The fifth root of x *added to* two thirds of *the fourth root of the third power of the square of a diminished by five* subtracted from *the square root of the difference between n and p.*

NUMERICAL VALUES.

16. 17. $x=\frac{(6\times1-3\times1)\times5\times4}{5\times4-3\times4}=7\frac{1}{2}$; $y=\frac{(5\times1-4\times1)\times3\times6}{5\times6-3\times4}=1.$

18. $x=\frac{(2+3)\times25}{2}=62\frac{1}{2}$; $y=\frac{(2+3)\times25}{3}=41\frac{2}{3}.$

19. $x=\frac{1}{2}\left(3+\sqrt{\frac{36-27}{9}}\right)=2$; $y=\frac{1}{2}\left(3-\sqrt{\frac{36-27}{9}}\right)=1.$

17. 20. $\frac{\frac{1}{4}+\frac{1}{512}}{\frac{1}{4}-\frac{1}{512}}=\frac{\frac{128}{512}+\frac{1}{512}}{\frac{128}{512}-\frac{1}{512}}=\frac{129}{127}$ *Ans.*

21. $\sqrt{\frac{16^{\frac{3}{4}}-8^{\frac{2}{3}}}{8^{\frac{1}{3}}-16^{\frac{1}{2}}}}=\sqrt{\frac{8-4}{\frac{1}{2}-\frac{1}{4}}}=\sqrt{\frac{4}{\frac{1}{4}}}=\sqrt{16}=4$ *Ans.*

22. $15\times2^{1+1}\times2^{1+1}\times2^{1+1}=15\times2^3\times2^3\times2^3=960$ *Ans.*

23. $(3+5)^0+2\times5^0\times7=1+2\times1\times7=15$ *Ans.*

ADDITION.

20. 15. $15x^2y$. 16. $2x^3y^4$.

17. $5x^2y^4$. 18. $28x^3y^3$.

20. 0.

22. 16. $-2xy+10$. 17. $6x^{\frac{3}{2}}-7y^{\frac{5}{3}}-6xy-10x^2y^2-3x$.

18. $-13c+8m-2n-5$. 19. $x+y+z$.

22. 20. $x^2+y^2+z^2$.
21. $10x^2+4xy$.
22. $2x^2+2y^2$.
23. $2x^3+6xy^2$.
24. $2x^4+12x^2y^2+2y^4$.
25. $2x^3+2y^2$.

23. 27. $2x+2y$.
28. $2x^6+30x^4y^2+30x^2y^4+2y^6$.

24. 7. *Ans.* 8.
9. *Ans.* $5x$.
10. *Ans.* $10x^4$.

SUBTRACTION.

26. 13. $2a^2x$.
14. $-31a^{\frac{1}{2}}x^{\frac{3}{4}}y$.
15. $42axyz$.
16. $11x^5$.
7. $3x-2y$.
8. $7x^3-2x^2$.
9. $10x+4m$.
10. $5xy-12$.
11. $12+4x$.
12. $15+2x^{\frac{1}{4}}$.

27. 17. $2xy$.
18. $x^5y+6x^4y^2+y^6$.
19. $-x^5y+6x^3y^4+y^6$.

28. 20. $5y-11$.
21. $2x-18$.
22. $-x^{\frac{3}{2}}+8x^{\frac{2}{3}}-7x^{\frac{1}{2}}$.
23. $2x+2x^3$.
24. $4x+10x^3$.
25. $10x$.
26. $2x^2+2x^3$.
27. $xy+y^2$.
28. y^3-x^2y.
29. $3x^5y+6x^4y^2+x^3y^3+3x^2y^4-y^6$.
3. $-10x-2xy+7y^2+19$.
4. x^2.
5. $6(x-y)+z+2$.
6. $(x-y)^{-2}+2$.

MULTIPLICATION.

33. 8. $56x^{\frac{3}{2}}y^{\frac{13}{12}}$. 9. $25x^{\frac{2}{3}}y^{\frac{19}{20}}$.

4. $-35xy^{\frac{5}{6}}$. 5. $-4mx$.

34. 7. $-24xyz$. 8. $-24ax^4y^4z$.

9. $-6abcxyz$. 3. $12a^4b^2x$.

4. axy. 5. $15x^2y^2z^2$.

6. $12axy^4$. 7. $40x^2y^2z^3$.

8. $14abxy$. 9. $45a^3bcxy$.

35. 5. $21x^5y^5z^5$. 6. $-22a^2y^2z^2$.

7. $-78m^2n^2p^2$. 8. $2xyz$.

9. $144x^4y^5z^6$. 10. $-169x^2y^2z^2$.

37. 12. x^3-x^2+3x+5. 13. $x^4+4x^3y+6x^2y^2+4xy^3+y^4$.

14. $x^4+ax^3+12x^2-2a^2x^2+3ax+35$.

15. $x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$.

16. $x^4-4x^3y^3+y^4$.

38. 20. $1-x^8$. 24. $x+\frac{2}{15}x^{\frac{7}{2}}-\frac{1}{15}x^6+\frac{8}{15}x^4-x^2$.

25. *Ans.* 70, Ex. 17. 26. $2x+\frac{5}{6}x^{\frac{5}{6}}+\frac{6}{5}x^{\frac{7}{6}}$.

29. *Ans.* 70, Ex. 28. 30. x^4-y^4.

31. x^8-y^8. 32. x^6-y^6

33. x^4-25x^2+49. 34. x^4-36x^2+324.

35. x^4-16x^2+64. 36. $x^4-4a^2x^2+4a^4$.

37. $x^4-4x^3-35x^2+78x+360$.

38 $x^3-15x^2+74x-120$.

39. 39. $x^2+10x+25$; $x^2-10x+25$; $x^2+11x+28$.

39. $x^2+3x-28$.

9. $x^4+2x^2y+y^2=5184$; $x^4+2x^2y^2+y^4=16384$.

9. $x^6+2x^4+x^2=270400$; $x^4+2x^3+x^2=5184$.

9. $x^{\frac{2}{3}}+2x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}=16$; $x^2+8x+16=144$.

9. $x^4+2x^5+x^6=331776$; $x^6+2x^7+x^8=21233664$.

40. 9. $x^2-6x+9=1$; $x^4-10x^2+25=121$.

9. $1-2x+x^2=9$; $9x^4-12x^3+4x^2=1600$.

9. $16x^6-16x^5+4x^4=4096$; $25x^4-20x^2y+4y^2=6084$.

9. $x^2-8x+16=0$.

9. (64.) $x^4-y^4=0$; $x^6-y^6=0$; $x^8-y^8=0$.

41. 7. $81x^4-108x^3+24x+4=13924$.

7. $x+y+z+2x^{\frac{1}{2}}y^{\frac{1}{2}}+2x^{\frac{1}{2}}z^{\frac{1}{2}}+2y^{\frac{1}{2}}z^{\frac{1}{2}}=81$.

7. $x^2+4y^2+9z^2+4xy+6xz+12yz=4900$.

8. $9x^2+24xy+16y^2=100$; $9x^2-24xy+16y^2=4$.

8. $9x^2-16y^2=20$.

8. $1+9x^2+16y^2+6x-8y-24xy=9$.

4.

x^4-a	x^3+ab	x^2-abc	$x+abcd$.
$-b$	$+ac$	$-abd$	
$-c$	$+ad$	$-acd$	
$-d$	$+bc$	$-bcd$	
	$+bd$		
	$+dc$		

DIVISION.

43. 9. $3ab$; $3a^2b$; $5abx$; $5ab^2x^2$.

10. $3a^2b^2c^2$; $-3x^4y^5$; $-2xy$; $3x^2y^2$.

11. $-\frac{1}{5}x^4y^3$; $-5x^2y^{-1}$; $\frac{3}{2}x^2b^{-1}$; $\frac{7}{8}a^{-1}mn$.

12. $2x^4$; *Ans.* 2; $-2y^2$; $2x^4y^4$.

17. $-27x^{\frac{2}{3}}y^{\frac{2}{3}}$; $15y^{\frac{1}{3}}$; $-2x^{\frac{1}{3}}y^{\frac{1}{3}}$.

18. $-21xy$; $-19xy$; $1\frac{3}{5}x^{-1}$.

5. $+2x^4-30x^3+3ax-2m+n$.

49. 9. *Ans.* 1; $x^5+x^4y+x^3y^2+x^2y^3+y^4$.

9. $x^6+x^5y+x^4y^2+x^3y^3+x^2y^4+xy^5+y^6$.

9. $x^7+x^6y+x^5y^2+x^4y^3+x^3y^4+x^2y^5+xy^6+y^7$.

9. *Ans.* 1.

9. $x^6-x^5y+x^4y^2-x^3y^3+x^2y^4-xy^5+y^6$.

9. $x^8-x^7y+x^6y^2-x^5y^3+x^4y^4-x^3y^5+x^2y^6-xy^7+y^8$.

50. 10. $x^7-x^6y+x^5y^2-x^4y^3+x^3y^4-x^2y^5+xy^6-y^7$.

10. $x^5-x^4y+x^3y^2-x^2y^3+xy^4-y^3$.

10. $x^6-x^5y+x^4y^2-x^3y^3+x^2y^4-xy^5+y^6-\dfrac{2y^7}{x+y}$.

10. $x^8-x^7y+x^6y^2-x^5y^3+x^4y^4-x^3y^5+x^2y^6-xy^7+y^8-\dfrac{2y^9}{x+y}$.

10. $x^6+x^5y+x^4y^2+x^3y^3+x^2y^4+xy^5+y^6+\dfrac{2y^7}{x-y}$.

10. $x^9-x^8y+x^7y^2-x^6y^3+x^5y^4-x^4y^5+x^3y^6-x^2y^7+xy^8-y^9+\dfrac{2y^{10}}{x+y}$.

12. $x+1$; x^2+x+1; $x^4-2x^3+4x^2-8x+16$.

12. x^2+3x+9; $x^4-3x^3+9x^2-27x+81$.

14. x^2+y^2; x^3+y^3; x^4+y^4; $x^9+x^6y^3+x^3y^6+y^9$.

FACTORING.

51. 2. $16a^3=2\times2\times2\times2\times a\times a\times a$, $169x^2y=13\times13xxxy$.

2. $112x^4y^2=2\times2\times2\times2\times7xxxxyy$.

2. $133x^2y^2=19\times7xxyy$.

4. $(a+b+c)\,x$; $(a^2+b^2)\,x^2$; $(5+2b)\,x$; $(x+y)\times3xy$.

5. $(x^2-xy+y^2)\,xy$; $(x+y)\,x^2y^2$; $(5a-2b+1)\,x$.

6. $(1+a-2b)\,x$; $(5b+1-a)\,x$; $(a+2b-4c)\times3x$.

3. $(8x^2-6x)^2=2\times2\times x\times x\times(4x-3)\,(4x-3)$.

3. $(1+x)\,(1+x)$; $(1-x)\,(1-x)$; $(1-x^{\frac{1}{3}})\,(1-x^{\frac{1}{3}})$.

4. $\left(1+\frac{a^{\frac{1}{2}}}{x^{\frac{1}{2}}}\right)\left(1+\frac{a^{\frac{1}{2}}}{x^{\frac{1}{2}}}\right)$; $\left(\frac{a}{x}-\frac{x}{a}\right)\left(\frac{a}{x}-\frac{x}{a}\right)$.

4. $(1+\frac{1}{2}x^{\frac{1}{3}})\,(1+\frac{1}{2}x^{\frac{1}{3}})$; $(1+\frac{1}{2x})\,(1+\frac{1}{2x})$.

5. $(x^{\frac{1}{2}}+y^{\frac{1}{2}})\,(x^{\frac{1}{2}}+y^{\frac{1}{2}})$; $(x^{\frac{1}{3}}-y^{\frac{1}{3}})\,(x^{\frac{1}{3}}-y^{\frac{1}{3}})$.

5. $(5x^{\frac{1}{2}}-2x)\,(5x^{\frac{1}{2}}-2x)$.

5. $(x^{\frac{1}{3}}-x^{\frac{2}{3}})\,(x^{\frac{1}{3}}-x^{\frac{2}{3}})=x^{\frac{1}{3}}\times x^{\frac{1}{3}}\,(1-x^{\frac{1}{3}})^2$.

6. $(1+\frac{1}{2x^{\frac{1}{2}}})\,(1+\frac{1}{2x^{\frac{1}{2}}})$; $(x-\frac{5}{2})\,(x-\frac{5}{2})$.

6. $(x+7)\,(x+7)$; $(x-\frac{3}{8})\,(x-\frac{3}{8})$.

53. 3. $(1+2x)\,(1-2x)$.

3. $(1-3x)\,(1+3x)$; $2\times2\times(1+2y)\,(1-2y)$.

3. $(3x+2y^2)\,(3x-2y^2)$; $(1-\frac{x}{2})\,(1+\frac{x}{2})$.

6. $x^8-y^8=(x^4+y^4)\,(x^2+y^2)\,(x+y)\,(x-y)$.

6. $x^{16}-y^{16}=(x^8+y^8)\,(x^4+y^4)\,(x^2+y^2)\,(x+y)\,(x-y)$.

6. $x^{32}-y^{32}=(x^{16}+y^{16})\,(x^8+y^8)\,(x^4+y^4)\,(x^2+y^2)\,(x+y)\,(x-y)$.

53. 6. $16x^4-16y^4=2\times2\times2\times2\times(x^2+y^2)\ (x+y)\ (x-y)$.

6. $16x^4-81y^4=(4x^2+9y^2)\ (2x+3y)\ (2x-3y)$.

7. $x-y=(x^{\frac{1}{2}}+y^{\frac{1}{2}})\ (x^{\frac{1}{2}}-y^{\frac{1}{2}})$.

7. $x^{\frac{2}{3}}-y^{\frac{2}{3}}=(x^{\frac{1}{3}}+y^{\frac{1}{3}})\ (x^{\frac{1}{3}}-y^{\frac{1}{3}})$.

7. $4x^3-y=(2x+y^{\frac{1}{2}})\ (2x-y^{\frac{1}{2}})$.

7. $x^{\frac{1}{2}}-y^{\frac{1}{2}}=(x^{\frac{1}{4}}+y^{\frac{1}{4}})\ (x^{\frac{1}{4}}-y^{\frac{1}{4}})$.

7. $x^{\frac{1}{3}}-y^{\frac{1}{3}}=(x^{\frac{1}{6}}+y^{\frac{1}{6}})\ (x^{\frac{1}{6}}-y^{\frac{1}{6}})$.

54. 5. $x^5+y^5=(x^4-x^3y+x^2y^2-xy^3+y^4)\ (x+y)$.

5. $x^5-y^5=(x^4+x^3y+x^2y^2+xy^3+y^4)\ (x-y)$.

5. $x^{10}-y^{10}=(x^5+y^5)\ (x^5-y^5)=(x^4-x^3y$, etc.$)\ (x+y)$
$(x^4+x^3y$, etc.$)\ (x-y)$.

5. $x^{12}-y^{12}=(x^6+y^6)\ (x^6-y^6)=(x^6+y^6)\ (x^3+y^3)\ (x^3-y^3)=$
$(x^6+y^6)\ (x+y)\ (x^2-xy+y^2)\ (x-y)\ (x^2+xy+y^2)$.

8. $x^{12}-y^{12}=(x-y)\ (x^{11}+x^{10}y+x^9y^2+x^8y^3+x^7y^4$, etc.$)$

$x^{12}-y^{12}=(x+y)\ (x^{11}-x^{10}y+x^9y^2-x^8y^3+x^7y^4$, etc.$)$

$x^{12}-y^{12}=(x^6+y^6)\ (x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5)\ (x-y)$.

$x^{12}-y^{12}=(x^6+y^6)\ (x^5-x^4y+x^3y^2+x^2y^3+xy^4-y^5)\ (x+y)$.

$x^{12}-y^{12}=$ *Vide* above, 54, 5.

8. $x^{16}-y^{16}=(x^{15}+x^{14}y+x^{13}y^2+x^{12}y^3+x^{11}y^4$, etc.$)\ (x-y)$.

$x^{16}-y^{16}=(x^{15}-x^{14}y+x^{13}y^2-x^{12}y^3+x^{11}y^4$, etc.$)\ (x+y)$.

$x^{16}-y^{16}=(x^{14}-x^{12}y^2+x^{10}y^4-x^8y^6$, etc.$)\ (x^2+y^2)$.

$x^{16}-y^{16}=(x^{12}-x^8y^4+x^4y^8-y^{12})\ (x^4+y^4)$.

$x^{16}-y^{16}=(x^{14}+x^{12}y^2+x^{10}y^4+x^8y^6$, etc.$)\ (x^2-y^2)$.

$x^{16}-y^{16}=(x^{12}+x^8y^4+x^4y^8+y^{12})\ (x^4-y^4)$.

$x^{16}-y^{16}=(x^8+y^8)\ (x^8-y^8)$ *Vide* Algebra, Ex. 7.

REMARK.—The teacher perceives that the above examples take an immense variety of forms. The pupil should be required to vary the answers without a moment's hesitation.

55. 5. $(x-7)(x-6)$; $(x+7)(x+6)$.

5. $(x-7)(x+6)$; $(x+7)(x-6)$.

6. $(x+4)(x-3)$; $(x-4)(x+3)$.

6. $(x-6)(x+5)$; $(x+20)(x-18)$.

7. $(x+8)(x+3)$; $(x^2-4)(x^2+1)=(x+2)(x-2)(x^2+1)$.

7. $x^4+10x^2+9=(x^2+9)(x^2+1)$.

7. $x^4-3x^2+2=(x^2-2)(x^2-1)=(x^2-2)(x+1)(x-1)$.

10. $x^4-37x^2+36=(x^2-36)(x^2-1)=(x+6)(x-6)(x+1)(x-1)$.

10. $x^8-26x^2+25=(x^3-25)(x^3-1)=(x^3-25)(x^2+x+1)(x-1)$.

10. $x^4-40x^2+144=(x^2-36)(x^2-4)=(x+6)(x-6)(x+2)(x-2)$.

12. $7x^2-7x-84=7(x-4)(x+3)$.

12. $5x^2-5x-60=5(x-4)(x+3)$.

12. $x^3-13x^2+42x=x(x-7)(x-6)$.

GREATEST COMMON DIVISOR.

56. 5. First *Ans.* $x+4$. Second *Ans.* $x+7$.

6. First *Ans.* $x+3$. Second *Ans.* x^2+xy+y^2.

7. First *Ans.* $x+2$. Second *Ans.* $x-5$.

58. 8.

OPERATION.

$x^3-10x^2+33x-36$	$3x^2-20x+33$
3	10
$2x^3-30x^2+99x-108$	$30x^2-200x+330$
$3x^3-20x^2+33x$	$30x^2-198x+324$
$-10x^2+66x-108$	$-2)-2x+6$
	Ans. $x-3$.

58. 9.

OPERATION.

$x^3-13x^2+56x-80$	$3x^2-26x+56$
3	13
$3x^3-39x^2+168x-240$	$39x^2-338x+728$
$3x^3-26x^2+56x$	$39x^2-336x+720$
$-13x^2+112x-240$	$-2)-2x+8$
	Ans. $x-4$.

FRACTIONS.

62. 2. First *Ans.* $\frac{x}{3y}$; Second *Ans.* $\frac{1}{x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}}}$; Third *Ans.* $\frac{x}{7y^{\frac{1}{2}}z^{\frac{2}{3}}}$.

4. (1.) $\frac{1+2y}{3x+4y}$. (2.) $\frac{x-y}{x+y}$. (3.) $\frac{x+y}{x-y}$.

6. $\frac{x^2-2x-35}{x^2+8x+15}=\frac{(x-7)(x+5)}{(x+7)(x+5)}=\frac{x-7}{x+3}$ *Ans.*

6. $\frac{x^2+15x+56}{x^2+5x-14}=\frac{(x+7)(x+8)}{(x+7)(x-2)}=\frac{x+8}{x-2}$ *Ans.*

6. $\frac{x^2-16}{x^2-x-20}=\frac{(x+4)(x-4)}{(x+4)(x-5)}=\frac{x-4}{x-5}$ *Ans.*

7. $\frac{x^2y-xy^2}{x^3y-2x^2y^2+xy^3}=\frac{xy(x^2-y^2)}{xy(x^2-2xy+y^2)}=\frac{x+y}{x-y}$ *Ans.*

7. $\frac{x^2-y^2}{x^4+x^2y^2+y^4}=\frac{(x^2+xy+y^2)(x-y)}{(x^2+xy+y^2)(x^2-xy+y^2)}=\frac{x-y}{x^2-xy+y^2}$.

7. $\frac{x^3-y^3}{x^4-y^4}=\frac{x^2+xy+y^2}{x^3+x^2y+xy^2+y^3}$ *Ans.*

9. $\frac{x^3-4x}{x^3-6x^2+8x}=\frac{x(x^2-4)}{x(x^2-6x+8)}=\frac{x+2}{x-4}$ *Ans.*

9. $\frac{x^4+2x^3-8x-16}{3(x^3-4)}=\frac{x^3-8}{3(x-2)}=\frac{x^2+2x+4}{3}$ *Ans.*

62. 9. $\dfrac{x^3+y^3}{x^4+x^3y^2+y^4}=\dfrac{x+y}{x^2+xy+y^2}$ *Ans.*

11. First *Ans.* $\dfrac{3x+5}{2x^2-7}$; Second *Ans.* $\dfrac{x+1}{x-1}$.

63. 6. $\dfrac{3x^2-6a^2}{3x}=\dfrac{x^2-2a^2}{x}=x-\dfrac{2a^2}{x}$ *Ans.*

6. $\dfrac{5x^2+10a^2}{5x}=\dfrac{x^2+2x^2}{x}=x+\dfrac{2a^2}{x}$ *Ans.*

6. $\dfrac{5x^2+7ax}{5x^2+3x}=\dfrac{5x+7a}{5x+3}=1+\dfrac{7a-3}{5x+3}$ *Ans.*

7. $\dfrac{x^2-y^2}{x-y}=x+y$ *Ans.* $\dfrac{x^3-y^3}{x-y}=x^2+xy+y^2$ *Ans.*

7. $\dfrac{x^4+y^4}{x+y}=x^3-x^2y+xy^2-y^3+\dfrac{2y^4}{x+y}$ *Ans.*

7. $\dfrac{x^4-y^4}{x+y}=x^3-x^2y+xy^2-y^3$ *Ans.*

8. $\dfrac{x^2+xy+y^2}{x+x^{\frac{1}{2}}y^{\frac{1}{2}}+y}=x-x^{\frac{1}{2}}y^{\frac{1}{2}}+y$ *Ans.*

8. $\dfrac{x^3+x^2y+xy^2+y^3}{x+y}=x^2+y^2$ *Ans.*

9. OPERATIONS.

(1.)	(2.)	(3.)
$\dfrac{x^2-4x}{x^3+6x^2+8x}$	$\dfrac{x^3+6x^2+8x}{x^3-4x}$	$\dfrac{x^2-2x-35}{x^2+8x+15}$
$\dfrac{x^2-4}{x^2+6x+8}$	$\dfrac{x^2+6x+8}{x^2-4}$	$\dfrac{(x-7)\,(x+5)}{(x+3)\,(x+5)}$
$\dfrac{(x+2)\,(x-2)}{(x+2)\,(x+4)}$	$\dfrac{(x+2)\,(x+4)}{(x+2)\,(x-2)}$	$\dfrac{x-7}{x+3}=1-\dfrac{10}{x+3}$ *Ans.*

$\dfrac{x-2}{x+4}=1-\dfrac{6}{x+4}$ *Ans.* $\dfrac{x+4}{x-2}=x+\dfrac{6}{x-2}$ *Ans.*

63. 11. $\frac{1+y^3}{1+x}=1-x+x^2-x^3+\frac{x^4+y^3}{1+x}$, or $1+y^3-y^3x+\frac{y^3x^3-x}{1+x}$.

11. $\frac{x^2-y^2}{x-1}=x+1+\frac{y^2+1}{x-1}$.

11. $\frac{1-y^2}{x-1}=\frac{y^2-1}{1-x}=y^2+y^2x+\frac{y^2x^2-1}{1-x}$. (*Vide* 89, (4.)

11. $\frac{1-x}{1+x}=1-2x+2x^2-2x^3+\frac{2x^4}{1+x}$.

64. 12. $\frac{x^4+x^2y^2+y^4}{x+y}=x^3-x^2y+2xy^2-2y^3+\frac{3y^4}{x+y}$.

12. $\frac{x^4-x^2y^2+y^4}{x-y}=x^3+x^2y+\frac{y^4}{x-y}$.

13. $\frac{3y^4}{y+x}=3y^3-3y^2x+3yx^2-3x^3+\frac{3x^4}{y+x}$.

13. $\frac{3x^4}{x+y}=3x^3-3x^2y+3xy^2-3y^3+\frac{3y^4}{x+y}$.

13. $\frac{x^3+5x^2+7x+3}{x^3+3x^2-x-3}=\frac{x+1}{x-1}=1+\frac{2}{x-1}$ *Ans.* (*Vide* 84, Ex. 2.)

6. $a-x+\frac{a^2+x^2}{a+x}=\frac{a^2-x^2+a^2+x^2}{a+x}=\frac{2a^2}{a+x}$ *Ans.*

6. $a-x-\frac{a^2+x^2}{a+x}=\frac{a^2-x^2-a^2-x^2}{a+x}=\frac{-2x^2}{a+x}$.

6. $x+y-\frac{x^2+y^2}{x+y}=\frac{x^2+2xy+y^2-x^2-y^2}{x+y}=\frac{2xy}{x+y}$ *Ans.*

7. $x+y-\frac{2xy}{x+y}=\frac{x^2+2xy+y^2-2xy}{x+y}=\frac{x^2+y^2}{x+y}$ *Ans.*

7. $x-y+\frac{x^2+y^2}{x-y}=\frac{x^2-2xy+y^2+x^2+y^2}{x-y}=\frac{2\,(x^2-xy+y^2)}{x-y}$.

7. $x-y+\frac{2xy}{x-y}=\frac{x^2-2xy+y^2+2xy}{x-y}=\frac{x^2+y^2}{x-y}$ *Ans.*

64. 8. $x^3+xy+y^2+\dfrac{x^3+y^3}{x-y}=\dfrac{x^3-y^3+x^3+y^3}{x-y}=\dfrac{2x^3}{x-y}$ *Ans.*

8. $x^3-xy+y^2+\dfrac{x^3-y^3}{x+y}=\dfrac{x^3+y^3+x^3-y^3}{x+y}=\dfrac{2x^3}{x+y}$ *Ans.*

65. 9. OPERATIONS.

(1.)	(2.)
$x^3-xy+y^2-\dfrac{x^2y^2}{x^2+xy+y^2}$	$x^3-xy+y^2-\dfrac{x^4+y^4}{x^2+xy+y^2}$
$\dfrac{x^4+x^2y^2+y^4-x^2y^2}{x^2+xy+y^2}$	$\dfrac{x^4+x^2y^2+y^4-x^4-y^4}{x^2+xy+y^2}$
$\dfrac{x^4+y^4}{x^2+xy+y^2}$ *Ans.*	$\dfrac{x^2y^2}{x^2+xy+y^2}$ *Ans.*

10. $1-x-\dfrac{1+x^2}{1+x}=\dfrac{1-x^2-1-x^2}{1+x}=\dfrac{-2x^2}{1+x}$ *Ans.*

10. $1+x+x^2-\dfrac{1+x^3}{1-x}=\dfrac{1-x^3-1-x^3}{1-x}=\dfrac{-2x^3}{1-x}$ *Ans.*

10. $1+x+x^2+\dfrac{1+x^3}{1-x}=\dfrac{1-x^3+1+x^3}{1-x}=\dfrac{2}{1-x}$ *Ans.*

11. $a+1-\dfrac{1+a+x}{2}=\dfrac{2a+2-1-a-x}{2}=\dfrac{a+1-x}{2}$ *Ans.*

11. $a+x-\dfrac{a^3-x^3}{a^2-ax+x^2}=a+x-a-x=0$ *Ans.*

11. $1+2a-\dfrac{3a+2a^2}{1+a}=\dfrac{1+3a+2a^2-3a-2a^2}{1+a}=\dfrac{1}{1+a}$ *Ans.*

65. 13. OPERATIONS.

(1.)

$$1-\frac{x^2+15x+56}{x^2+5x-14}$$

$$1-\frac{(x+7)(x+8)}{(x+7)(x-2)}$$

$$1-\frac{x+8}{x-2}$$

$$\frac{x-2-x-8}{x-2}=\frac{10}{2-x}\ \textit{Ans.}$$

(2.)

$$2+\frac{x^3+6x^2+8x}{x^3-4x}$$

$$2+\frac{(x+4)(x+2)}{(x+2)(x-2)}$$

$$2+\frac{x+4}{x-2}$$

$$\frac{2x-4+x+4}{x-2}=\frac{3x}{x-2}\ \textit{Ans.}$$

67. 3. $\frac{x}{11}+\frac{x}{13}+\frac{x}{26}+\frac{x}{52}=\frac{52x}{572}+\frac{44x}{572}+\frac{22x}{572}+\frac{11x}{572}=\frac{129x}{572}$ *Ans.*

4. $\frac{1}{x}+\frac{1}{y}=\frac{y}{xy}+\frac{x}{xy}=\frac{x+y}{xy}$ *Ans.*

4. $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{xy+xz+yz}{xyz}$ *Ans.* (*Vide* (IV), Ex. 4.)

4. $\frac{1}{x}+\frac{1}{x^2}=\frac{x}{x^2}+\frac{1}{x^2}=\frac{x+1}{x^2}$ *Ans.*

5. $\frac{1}{x^2}+\frac{1}{x^3}=\frac{x}{x^3}+\frac{1}{x^3}=\frac{x+1}{x^3}$ *Ans.*

5. $\frac{2}{x^2y}+\frac{3}{xy^2}+\frac{5}{x^2y^2}=\frac{2y+3x+5}{x^2y^2}$ *Ans.*

8. OPERATION.

$$\frac{1}{x^3-y^3}+\frac{x+y}{x^2+xy+y^2}+\frac{1}{x-y}$$

$$\frac{1}{x^3-y^3}+\frac{x^2-y^2}{x^3-y^3}+\frac{x^2+xy+y^2}{x^3-y^3}$$

$$\frac{1+xy+2x^2}{x^3-y^3}\ \textit{Ans.}$$

67. 9. OPERATION.

$$\frac{1}{x^4-y^4}+\frac{1}{x^3+x^2y+xy^2+y^3}+\frac{1}{x^2+y^2}+\frac{1}{x^2-y^2}$$

$$\frac{1}{x^4-y^4}+\frac{x-y}{x^4-y^4}+\frac{x^2-y^2}{x^4-y^4}+\frac{x^2+y^2}{x^4-y^4}$$

$$\frac{1+x-y+2x^2}{x^4-y^4} \quad Ans.$$

10. OPERATION.

$$\frac{1}{x+x^{\frac{1}{2}}y^{\frac{1}{2}}+y}+\frac{1}{x-x^{\frac{1}{2}}y^{\frac{1}{2}}+y}$$

$$\frac{x-x^{\frac{1}{2}}y^{\frac{1}{2}}+y}{x^2+xy+y^2}+\frac{x+x^{\frac{1}{2}}y^{\frac{1}{2}}+y}{x^2+xy+y^2}$$

$$\frac{2x+2y}{x^2+xy+y^2} \quad Ans.$$

OPERATIONS.

(12.)

$$\frac{x+3}{x-7}+\frac{x-7}{x+3}$$

$$\frac{x^2+6x+9}{x^2-4x-21}+\frac{x^2-14x+49}{x^2-4x-21}$$

$$\frac{2x^2-8x+58}{x^2-4x-21}$$

$$2+\frac{100}{x^2-4x-21} \quad Ans.$$

(13.)

$$\frac{x+4}{x-5}+\frac{x+5}{x-4}$$

$$\frac{x^2-16}{x^2-9x+20}+\frac{x^2-25}{x^2-9x+20}$$

$$\frac{2x^2-41}{x^2-9x+20}$$

$$2+\frac{18x-81}{x^2-9x+20} \quad Ans.$$

OPERATIONS.

68. (14.)

$$\frac{x^2+4x+3}{x^2+x-6}+\frac{x^2+8x+15}{x^2-25}$$

$$\frac{(x+3)\,(x+1)}{(x+3)\,(x-2)}+\frac{(x+5)\,(x+3)}{(x+5)\,(x-5)}$$

$$\frac{x+1}{x-2}+\frac{x+3}{x-5}$$

$$\frac{x^2-4x-5}{x^2-7x+10}+\frac{x^2+x-6}{x^2-7x+10}$$

$$\frac{2x^2-3x-11}{x^2-7x+10}$$

$$2+\frac{11x-31}{x^2-7x+10}\quad \textit{Ans.}$$

(15.)

$$\frac{x^2-1}{x^2+6x-7}+\frac{x^2-36}{x^2-2x-48}$$

$$\frac{(x+1)\,(x-1)}{(x+7)\,(x-1)}+\frac{(x+6)\,(x-6)}{(x-8)\,(x+6)}$$

$$\frac{x+1}{x+7}+\frac{x-6}{x-8}$$

$$\frac{x^2-7x-8}{x^2-x-56}+\frac{x^2+x-42}{x^2-x-56}$$

$$\frac{2x^2-6x-50}{x^2-x-56}$$

$$2-\frac{4x-62}{x^2-x-56}\quad \textit{Ans.}$$

68. 16. OPERATION.

$$\frac{x^2+5x+4}{x^2+3x-4}+\frac{x^2+4x-5}{x^2+6x+5}$$

$$\frac{(x+4)\,(x+1)}{(x+4)\,(x-1)}+\frac{(x+5)\,(x-1)}{(x+5)\,(x+1)}$$

$$\frac{x+1}{x-1}+\frac{x-1}{x+1}$$

$$\frac{x^2+2x+1}{x^2-1}+\frac{x^2-2x+1}{x^2-1}$$

$$\frac{2x^2+2}{x^2-1}=2+\frac{4}{x^2-1}\quad \textit{Ans.}$$

68. 17. OPERATION.

$$\frac{x^3+x^2+x+1}{x^3+x^2-x-1}+\frac{x^3-x^2-x+1}{x^3-x^2+x-1}$$

$$\frac{(x^2+1)\,(x+1)}{(x^2-1)\,(x+1)}+\frac{(x^2-1)\,(x-1)}{(x^2+1)\,(x-1)}$$

$$\frac{x^2+1}{x^2-1}+\frac{x^2-1}{x^2+1}$$

$$\frac{x^4+2x^2+1}{x^4-1}+\frac{x^4-2x^2+1}{x^4-1}$$

$$\frac{2x^4+2}{x^4-1}=2+\frac{4}{x^4-1}\quad \textit{Ans.}$$

OPERATIONS.

(18.)

$$\frac{x+y}{x-y}+\frac{x-y}{x+y}$$

$$\frac{x^2+2xy+y^2}{x^2-y^2}+\frac{x^2-2xy+y^2}{x^2-y^2}$$

$$\frac{2x^2+2y^2}{x^2-y^2}=2+\frac{4y^2}{x^2-y^2}\quad \textit{Ans.}$$

(19.)

$$\frac{x-y}{x^3-y^3}+\frac{x+y}{x^3+y^3}$$

$$\frac{1}{x^2+xy+y^2}+\frac{1}{x^2-xy+y^2}$$

$$\frac{2\,(x^2+y^2)}{x^4+x^2y^2+y^4}\quad \textit{Ans.}$$

20. OPERATION.

$$\frac{x-y}{x^5-y^5}+\frac{x+y}{x^5+y^5}$$

$$\frac{1}{x^4+x^3y+x^2y^2+xy^3+y^4}+\frac{1}{x^4-x^3y+x^2y^2-xy^3+y^4}$$

$$\frac{2\,(x^4+x^2y^2+y^4)}{x^8+x^6y^2+x^4y^4+x^2y^6+y^8}\quad \textit{Ans.}$$

or,

$$\frac{x^6+xy^5-x^5y-y^6}{x^{10}-y^{10}}+\frac{x^6-xy^5+x^5y-y^6}{x^{10}-y^{10}}$$

$$\frac{2\,(x^6-y^6)}{x^{10}-y^{10}}=\frac{2\,(x^4+x^2y^2+y^4)}{x^8+x^6y^2+x^4y^4+x^2y^6+y^8}\quad \textit{Ans.}$$

68. 21. $\frac{2x}{1-x^2}+\frac{1}{1+x}=\frac{2x}{1-x^2}+\frac{1-x}{1-x^2}=\frac{1+x}{1-x^2}=\frac{1}{1-x}$ *Ans.*

22. OPERATION.

$$\frac{2}{(x-1)^3}+\frac{3}{(x-1)^2}+\frac{4}{x-1}$$

$$\frac{2}{(x-1)^3}+\frac{3(x-1)}{(x-1)^3}+\frac{4(x-1)^2}{(x-1)^3}$$

$$\frac{4x^2-5x+3}{(x-1)^3}$$ *Ans.*

23. OPERATION.

$$\frac{x}{(1+x)(x+y)}+\frac{y}{(1-y)(x+y)}$$

$$\frac{x(1-y)}{(1-y)(1+x)(x+y)}+\frac{y(1+x)}{(1-y)(1+x)(x+y)}$$

$$\frac{x+y}{(1-y)(1+x)(x+y)}=\frac{1}{(1-y)(1+x)}$$ *Ans.*

24. OPERATION.

$$\frac{1}{4(1+x)}+\frac{1}{4(1-x)}+\frac{1}{2(1-x^2)}$$

$$\frac{1-x}{4(1-x^2)}+\frac{1+x}{4(1-x^2)}+\frac{2}{4(1-x^2)}$$

$$\frac{1}{1-x^2}$$ *Ans.*

25. *Ans.* $\frac{50x}{7}+\frac{50y}{7}-150.$

69. 5. $7x-\frac{4x}{5}=6x+\frac{x}{5}$ *Ans.* 6. $\frac{1}{x}-\frac{1}{x^2}=\frac{x-1}{x^2}$ *Ans.*

7. $\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{4x}{x^2-1}$ *Ans.* 8. $\frac{x+y}{x-y}-\frac{x-y}{x+y}=\frac{4xy}{x^2-y^2}$ *Ans.*

69. 9. OPERATION.

$$\frac{x+4}{x-6}-\frac{x-2}{x+3}$$

$$\frac{x^2+7x+12}{x^2-3x-18}-\frac{x^2-8x+12}{x^2-3x-18}$$

$$\frac{15x}{x^2-3x-9}\ \textit{Ans.}$$

10. OPERATION.

$$\frac{x^2+4x-12}{x^2+5x-14}-\frac{x^2+2x-15}{x^2+3x-18}$$

$$\frac{(x+6)\,(x-2)}{(x+7)\,(x-2)}-\frac{(x+5)\,(x-3)}{(x+6)\,(x-3)}$$

$$\frac{x+6}{x+7}-\frac{x+5}{x+6}=\frac{1}{x^2+13x+42}\ \textit{Ans.}$$

70. 11. OPERATION.

$$\frac{x^2-9}{x^2+x-12}-\frac{x^2+x-20}{x^2-25}$$

$$\frac{(x+3)\,(x-3)}{(x+4)\,(x-3)}-\frac{(x+5)\,(x-4)}{(x+5)\,(x-5)}$$

$$\frac{x+3}{x+4}-\frac{x-4}{x-5}=-\frac{2x-1}{x^2-x-20}\ \textit{Ans.}$$

12. *Ans.* $\frac{2xy}{x^4+x^2y^2+y^4}$. (*Vide* 68, 19.)

13. *Ans.* $\frac{4x^2}{x^4-1}$. (*Vide* 68, 17.)

14. *Ans.* $\frac{4xy}{x^2-y^2}$. (*Vide* 69, 8.)

15. *Ans.* $+\frac{3}{x^2+1}$.

70. 17. $m-\frac{bm+c}{a+b}=\frac{am+bm-bm-c}{a+b}=\frac{am-c}{a+b}$ *Ans.*

18. *Ans.* in order. $\frac{-ab+ac+bc}{2abc}$; $\frac{ab-ac+bc}{2abc}$; $\frac{ab+ac-b}{2abc}$.

19. *Ans.* $2b^3-b^2-\frac{1+7a}{12}$. 20. *Ans.* $x+\frac{32x+5}{105}$.

OPERATIONS.

71.

(5.)	(6.)
$2+\frac{2}{x-1}$ $\quad 2-\frac{2}{x+1}$	$2+\frac{2}{x-1}$ $\quad \frac{2}{x+1}$
$\frac{2x-2+2}{x-1}\times\frac{2x+2-2}{x+1}$	$\frac{2x-2+2}{x-1}\times\frac{2}{x+1}$
$\frac{4x^2}{x^2-1}$ *Ans.*	$\frac{4x}{x^2-1}$ *Ans.*

7. $\frac{x^2-y^2}{x^2+2xy+y^2}\times\frac{x+y}{x-y}=\frac{(x+y)\,(x-y)}{(x+y)\,(x+y)}\times\frac{x+y}{x-y}=1$ *Ans.*

8. $\frac{(x^2+y^2)\,(x+y)\,(x-y)}{(x-y)\,(x-y)}\times\frac{x-y}{x^2+y^2}=x+y$ *Ans.*

9. $\frac{(x^2-xy+y^2)\,(x+y)}{(x+y)\,(x-y)}\times\frac{x+y}{x^3-xy+y^2}=1+\frac{2y}{x-y}$ *Ans.*

10. $\frac{(x+y)\,(x^2-xy+y^2)\,(x-y)\,(x^2+xy+y^2)}{(x^2+y^2)\,(x+y)\,(x-y)}\times\frac{x^2+y^2}{x^3+xy+y^2}\times$

$\frac{x+y}{x^2-xy+y^2}=x+y$ *Ans.*

11. $\frac{(x-2)\,(x+1)}{(x-4)\,(x+3)}\times\frac{(x+5)\,(x-4)}{(x-7)\,(x+1)}=1+\frac{7x+11}{x^2-4x-21}$ *Ans.*

72. 12. $\frac{(x-3)\,(x+2)}{(x-5)\,(x+5)}\times\frac{(x-5)\,(x+3)}{(x-2)\,(x+2)}=1-\frac{3x-1}{x^2+3x-10}$ *Ans.*

13. $\frac{(x+2)\,(x+2)}{(x-3)\,(x-3)}\times\frac{(x+3)\,(x-3)}{(x-2)\,(x+2)}=1+\frac{10x}{x^2-5x+6}$ *Ans.*

72. 14. $\frac{(x-3)\,(x-3)}{(x+4)\,(x+4)}\times\frac{(x-4)\,(x+4)}{(x+3)\,(x-3)}=1-\frac{14x}{x^2+7x+12}$ *Ans.*

15. $\frac{(x-2a)\,(x+a)}{(x+3a)\,(x-a)}\times\frac{(x-7a)\,(x+3a)}{(x+3a)\,(x-2a)}=1-\frac{8ax+4a^2}{x^2+2ax-3a^2}$.

16. $\frac{a-x+a+x}{a-x}\times\frac{a+x-a+x}{a+x}=\frac{4ax}{a^2-x^2}$ *Ans.*

17. $\frac{1-x^2+2+x^2}{1-x}\times\frac{1-4x^2-3+4x^2}{1-2x}=\frac{-6}{1-3x+2x^2}$ *Ans.*

18. $\frac{a^2-x^2+a^2+x^2}{a+x}\times\frac{a^2-x^2-a^2-x^2}{a+x}=\frac{-4a^2x^2}{a^2+2ax+x^2}$ *Ans.*

19. OPERATION.

$$\left(x^2+xy+y^2+\frac{x^3+y^3}{x-y}\right)\times\left(x^2-xy+y^2+\frac{x^3-y^3}{x+y}\right)$$

$$\frac{x^3-y^3+x^3+y^3}{x-y}\times\frac{x^3+y^3+x^3-y^3}{x+y}$$

$$\frac{4x^6}{x^2-y^2}=4x^4+4x^2y^2+4y^4+\frac{4y^6}{x^2-y^2}$$ *Ans.*

20. OPERATION.

$$(x^2+x+1)\times\left(\frac{1}{x^2}-\frac{1}{x}+1\right)$$

$$(x^2+x+1)\times\frac{1-x+x^2}{x^2}$$

$$\frac{x^4+x^2+1}{x^2}=x^2+1+\frac{1}{x^2}$$ *Ans.*

21 *Ans.* $x^2+1+\frac{1}{x^2}$.

73. 4. *Ans.* $\frac{4}{x-y}$.

5. $\frac{a^2-x^2+a^2+x^2}{a+x}\times\frac{a+x}{a^2-x^2-a^2-x^2}=-\frac{a^2}{x^2}$ *Ans.*

6. $\frac{1-x^2-1-x^2}{1+x}\times\frac{1-x}{1-x^2-1-x^2}=\frac{1-x}{x(1+x)}$ *Ans.*

74. 14. $\frac{1-x^2+1+x^2}{1+x} \times \frac{1+x}{1-x^2-1-x^2} = -\frac{1}{x^2}$ *Ans.*

15. $\frac{1-x^2+1+x^2}{1-x} \times \frac{1+x}{1-x^2-1-x^2} = -\frac{1+x}{x^2(1-x)}$ *Ans.*

16. OPERATION.

$$\left(x^2-xy+y^2-\frac{x^2y^2}{x^2+xy+y^2}\right) \div \left(x^2-xy+y^2-\frac{x^4+y^4}{x^2+xy+y^2}\right)$$

$$\frac{x^4+x^2y^2+y^4-x^2y^2}{x^2+xy+y^2} \times \frac{x^2+xy+y^2}{x^4+x^2y^2+y^4-x^4-y^4}$$

$$\frac{x^4+y^4}{x^2y^2} = x^2y^{-2}+x^{-2}y^2 \quad \textit{Ans.}$$

17. *Ans.* $\frac{1}{2}(xy^{-1}+x^{-1}y)$. 18. *Ans.* $\frac{2x^{\frac{1}{2}}y^{\frac{1}{2}}}{x+y}$.

19. $\frac{2x^2+2y^2}{x^2-y^2} \times \frac{x^2-y^2}{4xy} = \frac{1}{2}(xy^{-1}+x^{-1}y)$ *Ans.* (*Vide* 70, 14, and 68, 18.)

20. *Ans.* $\frac{1}{2}(x^2y^{-2}+x^{-2}y^2)$.

22. *Ans.* $x^2+2y^2+\frac{3y^4}{x^2-y^2}$. (*Vide* 68, 18, 19.)

24. *Ans.* $\frac{4xy}{4x+2y+3}$. (*Vide* 70, 14, and 67, 7, Algebra.)

25. $\frac{2abc}{-ab+ac+bc}$ *Ans.* 26. $\frac{2abc}{ab-ac+bc}$ *Ans.*

27. $\frac{2abc}{ab+ac-bc}$ *Ans.*

EQUATIONS.

79. 4. $x=36$. 5. $x=25$.

80. 8. $x=26$. 9. $x=63$.

84. 4. $x=6$. 5. $x=60$.
6. $x=\frac{12}{13}$. 7. $x=15$.
16. $x=14$.

85. 25. $x=6$. 26. $x=19$.

PROBLEMS.

89. 4.* Let $20x=$ the number.
Then, $15x+8x=23$.
Whence, $x=1$, and $20x=20$ *Ans.*

6.* Let $6x=$ the number.
Then, $6x+3x+2x=11$.
Whence, $x=1$, and $6x=6$ *Ans.*

6.* Let $12x=$ the number.
Then, $6x+4x-3x=7$.
Whence, $x=1$, and $12x=12$ *Ans.*

7.* Let $45x=$ the number.
Then, $18x+20x-x=37$.
Whence, $x=1$, and $45x=45$ *Ans.*

89. 10.† Let $6x-11=$ the number.

Then, $\frac{4}{3}(6x-9)=\frac{3}{2}(6x-10)$ (1)

$8x-12=9x-15.$ (2)=(1) reduced.

Whence, $x=3$, and $6x-11=7$ *Ans.*

90. 12.* Let $6x=$ the number.

Then, $6x-3x-2x-3=1.$

Whence, $x=4$, and $6x=24$ *Ans.*

13.* Let $30x=$ the number.

Then, $6x-5x=4.$

Whence, $x=4$, and $30x=120$ *Ans.*

14.† Let $11x+2=$ the number.

Then, $x=6$, and $11x+2=68$ *Ans.*

15. Let $x=$ the number.

Then, $\frac{2x}{3}+\frac{3x}{4}=68$ (1)

$8x+9x=12\times 68$ (2)=(1)×12

$x=48$ *Ans.*

15.* Let $12x=$ the number.

Then, $8x+9x=68.$

Whence, $x=4$, and $12x=48$ *Ans.*

16. Let $x=$ the number.

Then, $\frac{x+4}{3}=5$ (1)

$x+4=15$ (2)=(1)×3

$x=11$ *Ans.*

16.† Let $3x-4=$ the number.

Then, $x=5$, and $3x-4=11$ *Ans.*

90. 17. Let x= the number.

Then, $\frac{2}{3}(x-3)=16$ (1)

$2x-6=48$ (2)=(1)×3

$x=27$ *Ans.*

17.† Let $3x+3$= the number.

Then, $2x=16$, $x=8$, and $3x+3=27$ *Ans.*

18.* Let $24x$= his sheep.

Then, there are $6x$ sheep in one flock, $4x$ in another, $3x$ in another, $2x$ in another, and 450 in another.

Whence, $6x+4x+3x+2x+450=24x$.

Hence, $9x=450$, $x=50$, and $24x=1200$

$6x=300$; $4x=200$; $3x=150$; $2x=100$.

19. Let x= the number.

Then, $x+10x=132$

$x=12$ *Ans.*

20. Let x= the price of the silver watch.

Then, $10x$= the price of the gold watch.

Hence, $11x=132$.

Whence, $x=\$12$, the price of the silver watch;

And $10x=\$120$, the price of the gold watch.

21. Let x= the price of the sheep.

Then, $12x$= the price of the cow;

And $24x$= the price of the ox.

Hence, $37x=74$.

Whence, $x=\$2$, price of the sheep;

$12x=\$24$, price of the cow;

And $24x=\$48$, price of the ox.

91. 22. Let x= the price of the key.

Then, $12x$= the price of the silver watch;

And $25x$= the price of the gold watch.

Hence, $38x$= 342.

Whence, x= \$9, price of key;

$12x$= \$108, price of silver watch;

And $25x$= \$225, price of gold watch.

23. Let x= price of 1 sheep;

$8x$= price of 1 cow;

$16x$= price of 1 ox.

Then, $20x$= price of 20 sheep;

$40x$= price of 5 cows;

$32x$= price of 2 oxen.

Hence, $92x$=460.

Whence, x= \$5, price of 1 sheep; $8x$= \$40, price of 1 cow; $16x$= \$80, price of 1 ox.

24. Let x= the number of days.

Since *one* man gets $\frac{1}{4}$\$ per day, *three* men will get $\frac{3}{4}$\$ for 1 day, and for x days they will get $\frac{3x}{4}$\$.

Since *one* boy gets $\frac{1}{5}$\$ per day, *two* boys will get $\frac{2}{5}$\$ for 1 day, and for x days they will get $\frac{2x}{5}$\$.

Hence, $\frac{3x}{4}+\frac{2x}{5}=23$.

Whence, x=20 days *Ans.*

24.* Let $20x$= the number of days.

Since *one* man gets $\frac{1}{4}$\$ per day, *three* men will get $\frac{3}{4}$\$ for 1 day, and for $20x$ days they will get $15x$\$.

Since *one* boy gets $\frac{1}{5}$\$ per day, *two* boys will get $\frac{2}{5}$\$ for 1 day, and for $20x$ days they will get $8x$\$.

Hence, $23x=23$.

Whence, $x=1$, and $20x=20$ days *Ans.*

92. 28. Let $x=$ the distance the 1st man travels.

Then, $\frac{3}{2}\times\frac{4}{7}\times x=\frac{6x}{7}=$ distance the 2d man travels.

Hence, $x+\frac{6x}{7}=520$.

Whence, $x=280$ feet, and $\frac{6x}{7}=240$ feet.

30. Let $x=$ the time.

Then, $\frac{3}{4}=$ the part the 1st will fill in one hour;

$\frac{3}{10}=$ the part the 2d will fill in one hour;

$\frac{1}{5}=$ the part the 3d will fill in one hour;

$\frac{1}{x}=$ the part all will fill in one hour.

Hence, (*axiom* 1,) $\frac{3}{4}+\frac{3}{10}+\frac{1}{5}=\frac{1}{x}$

Whence, $x=\frac{4}{5}$ of an hour $=48$ minutes *Ans.*

31. Let $x=$ the time.

Then, $\frac{1}{5}=$ the part the 1st will *fill* in one hour;

$\frac{1}{6}=$ the part the 2d will *fill* in one hour;

$\frac{7}{60}=$ the part the 3d mill *empty* in one hour;

$\frac{1}{x}=$ the part all will fill in one hour.

Hence, $\frac{1}{5}+\frac{1}{6}-\frac{7}{60}=\frac{1}{x}$

Whence, $x=4$ hours *Ans.*

93. 32. Let x = the time.

Then, $\frac{1}{3}$ = the part A can do in one day;

$\frac{1}{5}$ = the part B can do in one day;

$\frac{2}{15}$ = the part C can do in one day;

$\frac{1}{x}$ = the part A, B, and C can do in one day.

Hence, $\frac{1}{3}+\frac{1}{5}+\frac{2}{15}=\frac{1}{x}$; whence, $x=1\frac{1}{2}$ days *Ans.*

The equation for the figures in ( ) is,

$\frac{3}{8}+\frac{1}{2}+\frac{1}{8}=\frac{1}{x}$; whence, $x=1$ hour *Ans.*

33. Let x = the time in which B can do it alone.

Then, $\frac{1}{5}$ = the part A and B can do in one day;

$\frac{1}{7}$ = the part A alone can do in one day;

$\frac{1}{x}$ = the part B alone can do in one day.

Hence, $\frac{1}{5}-\frac{1}{7}=\frac{1}{x}$; whence, $x=17\frac{1}{2}$ days *Ans.*

The equation for the figures in ( ) is,

$\frac{11}{6}-\frac{3}{22}=\frac{1}{x}$; whence, $x=\frac{3}{5}\frac{3}{6}$ days *Ans.*

35. Let x = the time in which the man alone can drink it.

Then, $\frac{1}{x}$ = the part the man alone drinks in one day;

$\frac{1}{30}$ = the part the woman alone drinks in one day;

$\frac{1}{10}$ = the part both drink in one day.

Hence, $\frac{1}{10}-\frac{1}{30}=\frac{1}{x}$. Whence, $x=15$ days *Ans.*

93. 36. Let x = the time in which all can do the work.

Then, $\frac{1}{x}$ = the part A, B, C, and D can do in one day;

$\frac{3}{x}$ = " 3A, 3B, 3C, and 3D " "

$\frac{11}{6}$ = the part A, B, and C can do in one day;

$\frac{7}{4}$ = the part A, B, and D can do in one day;

$\frac{19}{12}$ = the part A, C, and D can do in one day;

$\frac{13}{12}$ = the part B, C, and D can do in one day;

$\frac{11}{6}+\frac{7}{4}+\frac{19}{12}+\frac{13}{12}$ = part 3A, 3B, 3C, and 3D can do in one day.

Hence, (*axiom* 1,) $\frac{11}{6}+\frac{7}{4}+\frac{19}{12}+\frac{13}{12}=\frac{3}{x}$

Whence, $x=1\frac{2}{5}$ days, in which all do the work.

Now, $\frac{1}{x}-\frac{13}{12}$ = the part A alone can do in one day $=\frac{1}{1\frac{2}{5}}$;

$\frac{1}{x}-\frac{19}{12}$ = the part B alone can do in one day $=\frac{6}{1\frac{2}{5}}$;

$\frac{1}{x}-\frac{7}{4}$ = the part C alone can do in one day $=\frac{4}{1\frac{2}{5}}$;

$\frac{1}{x}-\frac{11}{6}$ = the part D alone can do in one day $=\frac{3}{1\frac{2}{5}}$.

Therefore, A can do the work in 1 day, B in 2 days, C in 3 days, and D in 4 days.

93. 37. Let x = the less part.

Then, $55-x$ = the greater part;

And $55-2x$ = the difference of the parts.

Hence, $\frac{x}{55-2x}=2$.

Whence, $x=22$, the less part;

And $55-x=33$, the greater part.

93. 37. (2.) Let x= the greater part.

Then, $80-x$= the less part.

Hence, $\frac{x}{80}=\frac{3}{4}$

Whence, $x=60$, the greater part.

Ans. $80-x=20$, the less part.

38. Let x= the distance from A to B;

$\frac{2x}{3}$= the distance from C to D;

$\left(\frac{x}{4}+\frac{x}{3}\right)\times\frac{1}{3}=\frac{x}{12}+\frac{x}{9}$= the distance from B to C.

Hence, $x+\frac{2x}{3}+\frac{x}{12}+\frac{x}{9}=134$.

Whence, $x=72$ miles, distance from A to B;

$\frac{2x}{3}=48$ miles, distance from C to D;

$\frac{x}{12}+\frac{x}{9}=14$ miles, distance from B to C.

39. Let x= the money he had at first.

Then, $x-5$= money after spending 5 shillings;

$2x-10$

$3x-15$= money after borrowing;

$3x-20$= money after spending 5 shillings at 2d inn;

$6x-40$

$9x-60$= money after borrowing;

$9x-65$= money after spending 5 shillings at 3d inn;

$18x-130$

$27x-195$= money after borrowing;

$27x-216$= money after spending 21 shillings at 4th inn.

Hence, $27x-216=0$

Whence, $x=8$ shillings *Ans.*

94. 40. Let x = marbles he had at first.

Then, $x-2$ = marbles after laying aside 2;

$2x-4$ = marbles after winning;

$2x-7$ = marbles after laying aside 4;

$4x-14$ = marbles after winning;

$4x-18$ = marbles after laying aside 4;

$8x-36$ = marbles after winning.

Hence, $8x-36+9=13$.

Whence, $x=5$ marbles *Ans.*

42. Let x = his original stock.

Then, $x-1000$ = his gain the 1st year;

$2x-1000$ = stock at the end of the 1st year;

$2x-2000$ = gain the 2d year;

$4x-3000$ = stock at the end of the 2d year;

$4x-4000$ = gain the 3d year;

$8x-7000$ = stock at the end of the 3d year.

Hence, $8x-7000=3x$.

Whence, $x=\$1400$ *Ans.*

44. Let x = the less, and $40-x$ = the greater.

Then, $40x-x^2=35x$; whence 5 and 35 are the numbers.

45. Let x = the velocity per hour in 1st case.

Then, $\frac{2x}{3}$ = the velocity per hour in 2d case.

Now, $14-x$, or $4+\frac{2x}{3}$, is the velocity that would have been made by the man per hour in still water.

Hence, $14-x=4+\frac{2x}{3}$.

Whence, $x=6$ m., and $\frac{2x}{3}=4$ m.

94. 46. FIRST CASE.—Let x = the time.

Let M C represent the road traveled over, M being the point from which A and B started. By the first condition the parties will be situated as indicated by the letters.

Since A goes 7 miles per hour, in x hours he will go $7x$ miles = M A.

Since B goes 10 miles per hour, in x hours he will go $10x$ miles = M B.

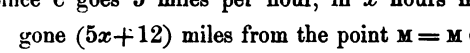
Since C goes 5 miles per hour, in x hours he will have gone $(5x+12)$ miles from the point M = M C.

Now, by the question, 2 M B = M A + M C.

Hence, $20x=7x+5x+12$.

Whence, $x=1\frac{1}{2}$ hours *Ans.*

46. SECOND CASE.—Let x = the time.



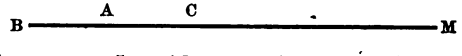
The parties will be situated as indicated by the letters.

Here, 2 M C = M A + M B.

Hence, $10x+24=7x+10x$.

Whence, $x=3\frac{3}{7}$ hours *Ans.*

46. THIRD CASE.—Let x = the time.



Here, M C = $5x-12$; M A = $7x$; M B = $10x$.

Now, 2 M A = M C + M B.

Hence, $14x=5x-12+10x$.

Whence, $x=12$ hours *Ans.*

95. 2. Let $20x$ = the rent last year.

Then, $20x+x=21x$ = the rent this year.

Hence, $21x=840$; $x=40$; $20x=\$800$ *Ans.*

ELIMINATION.

103. 2. $x=5$; $y=4$.
3. $x=10$; $y=20$.
4. $x=3$; $y=2$.
5. $x=1$; $y=1$.
6. $x=1$; $y=0$.
7. $x=11$; $y=7$.
8. $x=0$; $y=3$.
9. $x=\frac{1}{7}$; $y=\frac{1}{5}$.
10. $x=10$; $y=12$.
11. $x=14\frac{35}{149}$; $y=9\frac{393}{447}$.
12. $x=26$; $y=22$.
13. $x=56$; $y=40$.
14. $x=60$; $y=40$.
15. $x=17\frac{31}{37}$; $y=-\frac{7}{37}$.
16. $x=6$; $y=7$.
17. $x=1$; $y=3$.
18. $x=0$; $y=0$.
19. $x=12$; $y=48$.

104. 20. $x=2$; $y=3$.
21. $x=12$; $y=24$.
22. $x=3\frac{1}{2}$; $y=28$.
23. $x=7$; $y=14$.
24. $x=15$; $y=30$.
25. $x=49$; $y=90$.
26. $x=15$; $y=30$.
27. $x=30$; $y=14$.
28. $x=35$; $y=70$.
29. $x=0$; $y=1$.
30. $x=28$; $y=56$.
31. $x=1$; $y=1$.

106. 4. $x=2$; $y=3$.
5. $x=1$; $y=4$.
6. $x=12$; $y=16$.
7. $x=8$; $y=10$.
8. $x=10$; $y=20$.
9. $x=1$; $y=3$.
10. $x=5$; $y=4$.
11. $x=7$; $y=10$.
12. $x=8$; $y=4$.

107. 13. $x=6$; $y=12$.
14. $x=4$; $y=6$.
15. $x=3$; $y=5$.
20. $x=3$; $y=5$.
19. $x=2$; $y=3$.

109. 3. $x=1$; $y=1$. 4. $x=85$; $y=35$.
6. $x=80$; $y=50$. 7. $x=10\frac{3}{5}\frac{3}{9}$; $y=3\frac{5}{5}\frac{1}{9}$.
8. $x=24$; $y=8$. 9. $x=\frac{1}{2}\frac{1}{3}$; $y=\frac{7}{4}\frac{7}{6}$.
10. $x=2$; $y=4$. 11. $x=8$; $y=5$.
12. $x=\frac{1}{3}$; $y=\frac{1}{4}$.

111. 2. $x=4$; $y=6$; $z=8$. 3. $x=1$; $y=2$; $z=3$.
4. $x=24$; $y=9$; $z=5$. 5. $x=5$; $y=6$; $z=7$.

112. 6. $x=36$; $y=24$; $z=12$. 7. $x=4$; $y=6$; $z=2$.

113. 2. $x=1$; $y=2$; $z=3$; $w=4$. 3. $u=4$; $x=5$; $y=6$; $z=7$.

114. 2. $x=10$; $y=9$; $z=8$. 3. $x=3$; $y=4$; $z=5$.
4. $x=6$; $y=7$; $z=4$; $w=9$. 5. $x=12$; $y=8$; $z=6$; $w=4$
6. $x=3$; $y=4$; $z=5$.

PROBLEMS.

115. 3. Let $x=$ first number, $y=$ second.
Then, $3x+5y=165$, and $\frac{x}{4}+\frac{y}{7}=8$.
Whence, $x=20$, and $y=21$.

4. Let $x=$ first number, $y=$ second.
Then, $x+7=3y$, and $y+7=5x$.
Whence, $x=2$, and $y=3$.

5. Let $x=$ one, and $y=$ the other.
Then, $x+y=100$, and $x-y=20$.
Whence, $x=60$, and $y=40$.

115. 6. Let x= the greater, y= the less.
Then, $x+y=40$, and $x=3y$.
Whence, $x=30$, and $y=10$.

7. Let x= A's money, y= B's.
Then, $x+y=50$, and $x+5=y-5$.
Whence, $x=\$20$, and $y=\$30$.

8. Let x= value of 1st horse, y= value of 2d.
Then, $x+25=2y$, and $y+25=x-25$.
Whence, value of 1st horse, \$125; value of 2d, \$75.

116. 9. Let $6x$= the quantity of wheat and corn.
Then, $3x+5$= quantity of corn, and $2x+10$=wheat.
Hence, $3x+5+2x+10=6x$.
Whence, $x=15$.
$3x+5=50$ bushels corn; $2x+10=40$ bushels wheat.

10. Let x= 1st part, and $4x$= 2d part.
Then, $5x=50$; $x=10$, 1st part; and $4x=40$, 2d part.

11. Let $3x$= 1st part, and $5x$= 2d part.
Then, $8x=72$; $3x=27$, 1st part; $5x=45$, 2d part.

13. Let x=1st, y=2d, z=3d.
Then, $x+y=11$; $x+z=12$; $y+z=13$.
Whence, 5=1st, 6=2d, 7=3d.

14. Let x=1st, y=2d, z=3d.
Then, $\frac{1}{x}+\frac{1}{y}=5$; $\frac{1}{x}+\frac{1}{z}=7$; $\frac{1}{y}+\frac{1}{z}=8$.
Whence, $\frac{1}{2}$=1st, $\frac{1}{3}$=2d, $\frac{1}{5}$=3d.

116. 15. Let $5x$= the income of each.

Then, $4x$= what A spends per annum;

And $4x+50$= what B spends per annum;

25= debt of C per annum.

Hence, $4x+50=5x+25$.

Whence, $x=25$, and $5x=\$125$, the income of each.

16. Let $\frac{x}{y}$= the fraction.

Then, $\frac{x+8}{y}=2$; and $\frac{x}{y+8}=\frac{2}{3}$.

Whence, $2=12$, $y=10$, and the fraction is $\frac{12}{10}$.

117. 18 Let x= the left digit, y= the middle, z= the right.

Then, $100x+10y+z$= the number.

By the question, $y=2x$; (1)

$y=z+3$; (2)

And $100x+10y+z-99=100z+10y+x$. (3)

Whence, the number is 241 *Ans.*

19. Let u=4th, x=3d, y=2d, z=1st.

Then, $1000u+100x+10y+z$= the number.

By the question, $2u=y$; (1)

$z=x-2$; (2)

$100x=75y$; (3)

And $1000u+100x+10y+z-909=1000z+100y+10x+u$. (4)

Whence, the number is 2341.

20. Let x=3d, y=2d, z=1st.

Then, $100x+10y+z$= the number.

By the question, $x=2z$; (1)

$x+y+z=3$; (2)

And $100x+10y+z-81=100z+10x+y$. (3)

Whence, the number is 201 *Ans.*

118. 23. Let x = number of eagles, y = half-eagles.
Then, $x+y=560$, and $10x+5y=5000$.
Whence, he used 440 eagles and 120 half-eagles.

24. Let $2x$ = A's age now, and x = B's age.
Then, $2x-10=3x-30$.
Whence, A is now 40 and B 20 years of age.

25. Let $5x$ = A's, and $5y$ = B's property.
Then, $25x+y=2700$, and $25y+x=5100$.
Whence, A has \$500 and B \$1000.

26. Let $2x$ = value of gold, and y = value of silver watch.
Then, $y+50=x$, and $2x+50=5y$.
Whence, the gold watch is worth \$200; silver watch, \$50.

27. Let x = the majority votes, and $4y$ = the minority.
Then, $x-4y=80$, and $x+y+25=6y-50$.
Whence, the successful candidate got 700; the other 620.

28. Let x = the gold, y = the silver.
Since $19\frac{1}{2}\frac{6}{5}$ pounds of gold lose 1 pound when weighed in water, x pounds of gold will lose $\frac{25x}{491}$ pounds.
Since $10\frac{1}{2}$ pounds of silver lose 1 pound when weighed in water, y pounds of silver will lose $\frac{2y}{21}$ pounds.
Hence, $x+y=20$, and $\frac{25x}{491}+\frac{2y}{21}=\frac{5}{4}$.
Whence, $x=14.77$; $y=5.23$.

29. Let x = price of sherry per doz., y = brandy per doz
Then, $2x+y=234$, and $7x+8y=1185$.
Whence, the sherry is worth $76\frac{1}{3}$ s. per doz., brandy $81\frac{1}{3}$.

119. 30. Let $42x$ = the whole composition.

Then, $28x+10$ = niter; $7x-4\frac{1}{2}$ = sul.; $4x+\frac{10}{7}-2$ = char.

Hence, $39x+4\frac{1}{14}=42x$.

Whence, $3x=4\frac{1}{14}$, and $42x=69$ pounds *Ans.*

31. Let x = first price of sherry, and y = port per doz.

Then, $x+1$ = price of sherry after rising, and $y+1$ = port.

Hence, $30x+20y=120$;

And $30y+30+25x+25=195$.

Whence, sherry was worth \$2; port \$3 per dozen.

32. Let x = A's, y = B's, z = C's property.

Then, $x+\frac{1}{2}(y+z)=5500$; $y+\frac{1}{3}(x+z)=5000$; $z+\frac{1}{4}(x+y)=5250$.

Whence, A had \$2000, B \$3000, and C \$4000.

33. Let x = the time past noon.

Then, $12-x$ = the time till midnight.

Hence, $\frac{x}{2}=\frac{60-5x}{26}$.

Whence, $x=3\frac{1}{3}$, and it is 20 minutes past 3 o'clock.

34. 200 bushels, at 80 cents per bushel, will bring \$160 00;
30 bushels, at 30 cents per bushel, will bring 9 00.

Hence, he must have 170 bushels of corn and barley in the mixture, worth \$151.00.

Let x = corn, and y = barley.

Then, $x+y=170$, and $70x+90y=15100$.

Whence, he must use 10 bushels of corn, and 160 of barley.

35. 136 bushels, at 48 pence per bu., will bring 6528 d.;
86 bushels, at 54 pence per bu., will bring 4644 d.

Hence, he must have 50 bushels barley and rye in the mixture, worth 1884 pence.

Let x= barley, and y= rye.

Then, $x+y=50$, and $36x+42y=1884$.

Therefore, he must use 36 bu. of barley and 14 of rye.

119. 36. Let u= share of 1st, x=2d, y=3d, z=4th, and m= the whole amount divided.

Then, $u+x+y+z=m$; (1)

And $u=\frac{m-u}{2}$ (2) $x=\frac{m-x}{3}$ (3) $y=\frac{m-y}{4}$ (4) $u=z+14$. (5)

Whence, $u=\frac{m}{3}$ (6) $x=\frac{m}{4}$ (7) $y=\frac{m}{5}$; (8)

$$\frac{m}{3}+\frac{m}{4}+\frac{m}{5}+z=m. \qquad (9)=(1)$$

Hence, $z=\frac{13m}{60}$ (10) and $u=\frac{13m}{60}+14$. (11)=(5)

Then, $\frac{m}{3}=\frac{13m}{60}+14$. (12)=(11) & (6) compared.

The whole amount divided was then \$120.

The 1st share was \$40; 2d, \$30; 3d, \$24; 4th, \$26.

120. 37. Let $11x$= what he owed to the 1st creditor;

$18y$= what he owed to the 2d creditor.

Then, $4x+3y+3=53$, and $3x+5y-1=42$.

Hence, he owed the first \$121, and the second \$36.

38. Let x= what A had, y= what B had, z= what C had.

After the first division the parties had,

$x-y-z$; $2y$; $2z$.

After the second division the parties had,

$2x-2y-2z$; $3y-x-z$; $4z$.

After the third division the parties had,

$4x-4y-4z=32$; $6y-2x-2z=32$; $7z-x-y=32$.

Whence, A had \$52; B, \$28; C, \$16, at first.

120. 39. Let x= dimes, and y= number of half-dollars.
Then, $x+y=6$, and $10x+50y=100$.
Therefore, *one* half-dollar was used, and *five* dimes.

40. Let x= number of dimes, y= half-dollars.
Then, $x+y=12$, and $10x+50y=200$.
Therefore, he used 10 dimes and 2 half-dollars.

41. Let x= the number of eagles, y= half-eagles.
Then, $x+y=8$, and $10x+5y=65$.
Therefore, he uses 5 eagles and 3 half-eagles.

42. Let x= the number of dimes, y=half-dimes.
Then, $x+y=73$, and $10x+5y=615$.
And he uses 50 dimes and 23 half-dimes.

INVOLUTION.

134. 5. $4x^2$; $9x^4y^2$; $16x^2y^4$; $25x^4y^4$; $36x^6y^2$; $49x^2y^6$; $64x^8y^8$.

6. $8x^6$; $125x^3y^9$; $343x^9y^3$; $512x^6y^9$; $729x^9y^{15}$; $1000x^3y^{18}$; $1728x^6y^6z^6$.

15. x^{10}; $x^{10}y^2$; $9x^2y^4$; $16x^6y^2$; $25x^2y^6$; xy; $x^{\frac{1}{2}}y^{\frac{1}{2}}$; x^6.

16. $-x^5$; x^9y; $125x^{\frac{1}{3}}y^{27}$; $-xy^6$; $-343x^3y^{\frac{2}{3}}$; $512x^4y^2$; and $-x^{21}$.

135. 4. $\frac{16x^4}{81}$; $\frac{x}{16y^4}$; $\frac{x^{11}}{16}$; $\frac{16y^8}{x^3}$.

5. $\frac{x^6}{y}$; $\frac{x^6}{64}$; $\frac{4096x^3}{15625y^3}$.

REMARK.—In the application of the Binomial Theorem the following table of coefficients will be valuable:

					1						$=(x+y)^0$
				1		1					
			1		2		1				
		1		3		3		1			
	1		4		6		4		1		
1		5		10		10		5		1	
	1		6		15		20		15		6		1
		1		7		21		35		35		21		7		1	
	1		8		28		56		70		56		28		8		1	
1		9		36		84		126		126		84		36		9		1
1	10		45		120		210		252		210		120		45		10		1

139. 1.

OPERATION.

1	5	10	10	5	1
$243x^5$	$81x^4$	$27x^3$	$9x^2$	$3x$	
	$2y$	$4y^2$	$8y^3$	$16y^4$	$32y^5$

$(3x+2y)^5=243x^5+810x^4y+1080x^3y^2+720x^2y^3+240xy^4+32y^5$

(1) The first line contains the binomial coefficients of $(x+y)^5$.

(2) The second line contains the decreasing powers of $3x$.

(3) The third line contains the increasing powers of $2x$.

(4) The fourth line contains the products of the corresponding terms of the other three lines.

4.

OPERATION.

	1	3	3	
	$8x^6$	$4x^4$	$2x^2$	
		$-3y^3$	$9y^6$	$-27y^9$
$(2x^2-3x^3)^3=$	$8x^6$	$-36x^4y^3$	$+54x^2y^6$	$-27y^9$

LOGARITHMS.

150.

Logarithm of .015364 = $\bar{2}$.186504.
Logarithm of 123456 = 5.091512.
Logarithm of .023967 = $\bar{2}$.379614.
Logarithm of .111122 = $\bar{1}$.045800.
Logarithm of .999999 = $\bar{0}$.000000.

(1.)

Logarithm of 6.832 = 0.834548.
Logarithm of .0362 = $\bar{2}$.558709.
Logarithm of 188.725 = 2.275839.

(2.)

Logarithm of .00634 = $\bar{3}$.802089.
Logarithm of 62.18 = 1.793651.
Logarithm of .00010196 = $\bar{4}$.008438.

(3.)

Logarithm of 3642 = 3.561340.
Logarithm of 23.68 = 1.374382.
Logarithm of 153.8 = 2.186958.

(4.)

Logarithm of .657 = $\bar{1}$.817565.
Logarithm of .0793 = $\bar{2}$.899273.
Logarithm of 8.285 = 0.918292.

RADICALS.

160. 6. $2\sqrt{19}$; $3\sqrt{6}$; $32\sqrt{2}$; $2\sqrt{21}$; $3\sqrt{21}$; $4\sqrt{30}$; $13\sqrt{2}$.

7. $14\sqrt{2}$; $15\sqrt{3}$; $16\sqrt{5}$; $17\sqrt{7}$; $18\sqrt{11}$; $19\sqrt{13}$.

9. $3\sqrt[3]{3}$; $3\sqrt[3]{5}$; $3\sqrt[3]{7}$; $3\sqrt[3]{11}$; $3\sqrt[3]{13}$.

10. $4\sqrt[3]{5}$; $4\sqrt[3]{7}$; $4\sqrt[3]{11}$; $5\sqrt[3]{9}$; $6\sqrt[3]{11}$.

161. 13. $4\sqrt{2} = 5.65685$; $5\sqrt{2} = 7.07106$; $6\sqrt{2} = 8.48528$.
$8\sqrt{2} = 11.31371$; $9\sqrt{2} = 12.72792$; $10\sqrt{2} = 14.14214$.

162. 3. $19xy^2$; $21xy^{10}$; $16x^2y^4$.

4. $18xy^6$; $20x^2y^7$; $22x^4y^6$.

5. $2xy^2$; $8x^3y^6$; $2xy^2$.

6. $3xy^2$; $2xy^2$; $3x^2y^3$.

163. 3. $2xy^2$; $-3x^2y$; $13x^2y^4$.

4. xy^2; $2xy^2$; $3ab$.

5. *Imaginary;* $-3x^2y^3$; $23xy^2$.

6. x^2y^4; $4x^2y^4$; $-6xy^2$.

CASE V.

3. $2xy^2\sqrt[4]{2}$; $2x\sqrt[6]{3x^2y^5}$; $\frac{1}{3}x^2y^2\sqrt{6y}$; $\frac{1}{2}xy\sqrt[3]{4y}$.

4. $\frac{1}{3}x^2y^3\sqrt{5}$; $\frac{3}{10}x^2y\sqrt{10}$; $2x^3y^4\sqrt{15}$; $2xy\sqrt[3]{2y}$.

5. $4x^3y^2\sqrt[3]{2}$; $2x^2y^3\sqrt[5]{2}$; $\frac{1}{2}xy^2\sqrt{2x}$; $\frac{3}{5}x^2y^4\sqrt{5y}$.

6. $\frac{1}{5}xy^2\sqrt[5]{250}$; $\frac{1}{5}x^4y^6\sqrt{10}$; $x^2y^3\sqrt{10xy}$; $30x^2y^2\sqrt{2y}$.

7. $2x^2y^2\sqrt{11y}$; $5x^2y^3\sqrt{3xy}$; $2x^2y^5\sqrt{2y}$; $14x^2y^3\sqrt{7x}$.

8. $5x^3y^4\sqrt{2x}$; $10x^2y^3\sqrt{2y}$; $9y^5\sqrt{3x}$; $\frac{1}{2}x^4y^5\sqrt{2x}$.

9. $\frac{1}{2}x^4y^4\sqrt{y}$; $\frac{1}{7}x^2y^2\sqrt{14y}$; $\frac{1}{4}y^7\sqrt{6xy}$; $z^7\sqrt{xyz}$.

164. 5. $14xy^2\sqrt{11xy}$.

6. $5\frac{12}{25}xy^2\sqrt{35}$.

7. $1\frac{1}{8}xy^3\sqrt{6}$.

8. $\frac{12}{10}\frac{7}{5}xy^2\sqrt{21y}$.

164. 9. $\frac{107}{30}xy^2\sqrt{5}$. 10. $14xy^2\sqrt[3]{5}$.

11. $14xy^2\sqrt[4]{2}$. 12. $\frac{1}{4}(3\sqrt[3]{4}+2\sqrt[3]{4y})xy^2$

165. 13. $\frac{7}{3}(\sqrt{2}+\sqrt{3}+\sqrt{y})xy^2$.

14. $(\frac{1}{7}xy^2+\frac{1}{2}x^2y^3+\frac{1}{3}x^2y^2)\sqrt{7}$.

15. $6(\sqrt[3]{3}+\sqrt[3]{4}+\sqrt[3]{5})xy^2$.

16. $6(\sqrt{5}+\sqrt{7}+\sqrt{11}+1)x^2y^3$.

17. $\frac{71}{105}(\sqrt{3}+\sqrt{5}+\sqrt{7})xyz$. 18. $2(\sqrt{2}+12)xy^2$.

4. $2x^2y\sqrt{5}$. 5. $\frac{1}{5}xy^3\sqrt{5}$.

6. $\frac{1}{3}\frac{1}{0}xy^2\sqrt{30}$. 7. $-\frac{9}{38}xy^3\sqrt{7}$.

8. $\frac{2}{3}x^3y^3\sqrt{3}$. 9. xy^3.

10. $x^4y^5\sqrt[3]{3}$.

166. 11. xy^2. 12. $xy^2\sqrt[5]{y}$.

13. $(2\sqrt[6]{y}-1)xy^2$. 14. $\frac{1}{2}\frac{1}{9}x^2y\sqrt{58}$.

15. $\frac{1}{4}\frac{1}{2}x^2y^2\sqrt{2y}$. 16. *Ans.* 0.

17. *Ans.* $\frac{2}{9}\sqrt{6}$.

167. 11. $\sqrt{x^4-a^4}$, 12. $\sqrt{x^2-y}$.

13. *Ans.* $\sqrt{5}$. 14. *Ans.* 5.

15. $5+\sqrt{2}$. 16. $5x-4y$.

17. *Ans.* 4. 18. $17+2\sqrt{30}$.

19. $2x+3y+2\sqrt{6xy}$. 20. *Ans.* 32.

21. $5x+8y+4\sqrt{10xy}$. 22. $80x+108\sqrt{xy}-126y$.

23. $90x-90y+122\sqrt{xy}$. 24. $x-y$.

25. $\dfrac{x+11\sqrt{x+28}}{x-11\sqrt{x+28}}$. 26. $\dfrac{x-25}{x-49}$.

32. $x-9$; $x-4$; $x-16$.

169. 4. $\dfrac{5}{7-\sqrt{40}}=\dfrac{35+10\sqrt{10}}{9}=7.40253.$

$\dfrac{8}{\sqrt{3}-\sqrt{7}}=-2(\sqrt{3}+\sqrt{7})=-8.7556.$

$\dfrac{5}{9-\sqrt{8}}=\dfrac{45+10\sqrt{2}}{73}=.810166.$

$\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}=5+2\sqrt{6}=9.898979.$

5. $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}=5-2\sqrt{6}=.1010206.$

$\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=4+\sqrt{15}=7.8729833.$

$\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=4-\sqrt{15}=.1270167.$

7. $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}=6+\sqrt{35}=11.9160798.$

$\dfrac{2\sqrt{11}-3\sqrt{13}}{2\sqrt{11}+3\sqrt{13}}=\dfrac{12\sqrt{143}-161}{73}=-.239738.$

EQUATIONS OF THE SECOND DEGREE.

180. 9. Let x = the whole quantity drawn out each time.

Now, a = the wine in the cask at first.

Then, $a-x$ = the wine left after the 1st drawing;

And x = water poured in after 1st drawing.

He now draws out x gallons of the mixture, and the quantity of wine in this x gallons is to the quantity of water as the wine left after the first drawing is to the water poured in.

Hence, (*vide* Arithmetic, 263,)

$$\frac{x(a-x)}{a} = \text{wine in 2d drawing, and } \frac{x^2}{a} = \text{water.}$$

Then, $a-x-\frac{x(a-x)}{a} = \frac{(a-x)^2}{a} =$ wine left after 2d drawing;

And $2x-\frac{x^2}{a} = \frac{x(a-x)}{a} =$ the water in the cask, including that poured in after 2d drawing.

In the same way,

$$\frac{x(a-x)^2}{a^2} = \text{wine in 3d drawing, and } \frac{x^2(2a-x)}{a^2} = \text{water.}$$

Then, $\frac{(a-x)^2}{a} - \frac{x(a-x)^2}{a^2} = \frac{(a-x)^3}{a^2} =$ wine after 3d drawing;

And $3x-\frac{x^2}{a}-\frac{2ax^2-x^3}{a^2} = \frac{3ax(a-x)+x^3}{a^2} =$ water in the cask, including that poured in after 3d drawing.

Again, $\frac{x(a-x)^3}{a^3} =$ wine in 4th draw., and $\frac{3ax^2(a-x)+x^4}{a^3} =$ water.

Hence, $\frac{(a-x)^3}{a^2} - \frac{x(a-x)^3}{a^3} = \frac{(a-x)^4}{a^3} =$ wine after 4th drawing;

And $\frac{2ax(a-x)^2+x^2(2a^2-x^2)}{a^3}$ = water in the cask, including that poured in after 4th drawing.

Whence, $\frac{(a-x)^4}{a^3}=b$, or $\frac{a-x}{a^{\frac{3}{4}}}=b^{\frac{1}{4}}$, and $x=a^{\frac{3}{4}}(a^{\frac{1}{4}}-b^{\frac{1}{4}})$.

$\frac{x(a-x)}{a}=a^{\frac{2}{4}}b^{\frac{1}{4}}(a^{\frac{1}{4}}-b^{\frac{1}{4}})$; $\frac{x(a-x)^2}{a^2}=a^{\frac{1}{4}}b^{\frac{2}{4}}(a^{\frac{1}{4}}-b^{\frac{1}{4}})$.

$\frac{x(a-x)^3}{a^3}=b^{\frac{3}{4}}(a^{\frac{1}{4}}-b^{\frac{1}{4}})$.

RADICAL EQUATIONS.

193. 10. $17+2\sqrt{x^2+9}=27$ (1)

$\sqrt{x^2+9}=5$ (2)

Hence, $x^2+9=25$, and $x=\pm 4$.

11. $5-\sqrt{25-x^2}=3x$ (1)

$25-x^2=9x^2-30x+25$ (2)

Hence, $10x=30$, and $x=3$.

12. $\sqrt{x-32}=16-\sqrt{x}$ (1)

$x-32=256-32\sqrt{x}+x$ (2)=(1)^2

Hence, $\sqrt{x}=9$, and $x=81$.

13. $\sqrt{x+40}=10-\sqrt{x}$ (1)

$x+40=100-20\sqrt{x}+x$ (2)=(1)^2

Hence, $\sqrt{x}=3$, and $x=9$.

14. $\sqrt{x-16}=\sqrt{x}-2$ (1)

$x-16=x-4\sqrt{x}+4$ (2)=(1)^2

Hence, $\sqrt{x}=5$, and $x=25$.

193. 15. $\sqrt{x+8}-\sqrt{x-8}=2\sqrt{2}$ (1)

$x+8-2\sqrt{x^2-64}+x-8=8$ (2)=(1)^2

$\sqrt{x^2-64}=x-4$ (3)=(2) reduced.

$x^2-64=x^2-8x+16$ (4)=(3)^2

Hence, $8x=80$, and $x=10$.

16. $\sqrt{x}+\sqrt{x-9}=\dfrac{9}{\sqrt{x-9}}$ (1)

$\sqrt{x^2-9x}+x-9=9$ (2)=(1)$\times\sqrt{x-9}$

$\sqrt{x^2-9x}=18-x$ (3)=(2)

$x^2-9x=324-36x+x^2$ (4)=(3)^2

Hence, $27x=324$, and $x=12$.

17. $\sqrt{1+x\sqrt{x^2-1}}=1-x$ (1)

$1+x\sqrt{x^2-1}=1-2x+x^2$ (2)=(1)^2

$\sqrt{x^2-1}=x-2$ (3)=(2) reduced.

$x^2-1=x^2-4x+4$ (4)=(3)^2

Hence, $4x=5$, and $x=\frac{5}{4}$.

18. $\sqrt{x-\sqrt{10-x}}=\dfrac{\sqrt{x}+\sqrt{10-x}}{2}$ (1)

$\sqrt{x}=3\sqrt{10-x}$ (2)=(1) reduced.

$x=90-9x$ (3)=(2)^2

Hence, $x=9$.

19. $\dfrac{9x-1}{\sqrt{9x}+1}=4+\dfrac{\sqrt{9x}-1}{2}$ (1)

$18x-2=24\sqrt{x}+8+9x-1$ (2)=(1) cleared of [fractions.

$x-\frac{8}{3}\sqrt{x}=1$ (3)=(2) reduced.

Hence, $\sqrt{x}=3$, and $\sqrt{x}=-\frac{1}{3}$.

REMARK.—x may be 9, or $\frac{1}{9}$, with the preceding limitations.

193. 20. $\dfrac{3\sqrt{2x}+10}{3\sqrt{2x}-10}=\dfrac{\sqrt{2x}+16}{\sqrt{2x}-4}$ (1)

$6x-2\sqrt{2x}-40=6x+38\sqrt{2x}-160$ (2)=(1) cleared of [fractions.

$\sqrt{2x}=3$ (3)=(2) reduced.

Hence, $2x=9$, and $x=4\frac{1}{2}$.

21. $\dfrac{\sqrt{x}+28}{\sqrt{x}+4}=\dfrac{\sqrt{x}+38}{\sqrt{x}+6}$ (1)

$x+34\sqrt{x}+168=x+42\sqrt{x}+152$ (2)=(1) cleared of [fractions.

Hence, $\sqrt{x}=2$, and $x=4$.

22. $\dfrac{a}{x}+\dfrac{\sqrt{a^2-x^2}}{x}=\dfrac{x}{b}$ (1)

$\dfrac{\sqrt{a^2-x^2}}{x}=\dfrac{x}{b}-\dfrac{a}{x}$ (2)=(1) transposed.

$\dfrac{a^2-x^2}{x^2}=\dfrac{x^2}{b^2}-\dfrac{2a}{b}+\dfrac{a^2}{x^2}$ (3)=(2)^2

Hence, $x^2=2ab-b^2$, and $x=\pm\sqrt{2ab-b^2}$.

23. $\dfrac{2\sqrt{x}+a}{2a+\sqrt{x}}=\dfrac{2a-\sqrt{x}}{\sqrt{x}}$ (1)

$2x+a\sqrt{x}=4a^2-x$ (2)=(1) cleared of [fractions.

$x+\frac{a}{3}\sqrt{x}=\dfrac{4a^2}{3}$

Hence, $\sqrt{x}=a$, and $\sqrt{x}=-\dfrac{4a}{3}$; and, with these limitations, $x=a^2$, or $\dfrac{16a^2}{9}$.

24. $\dfrac{a-\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}=b$ (1)

$\dfrac{(a-\sqrt{a^2-x^2})^2}{x^2}=b$ (2) (*Vide* Algebra, 164, Ex. 8.)

$$a-\sqrt{a^2-x^2}=\pm x\sqrt{b} \qquad (3)=\sqrt{(2)}$$

$$-\sqrt{a^2-x^2}=\pm x\sqrt{b}-a \qquad (4)=(3) \text{ transposed.}$$

$$a^2-x^2=x^2b\mp 2ax\sqrt{b}+a^2 \qquad (5)=(4)^2$$

$$x(1+b)=\pm 2a\sqrt{b} \qquad (6)=(5) \text{ reduced.}$$

Hence, $$x=\pm\frac{2a\sqrt{b}}{1+b}.$$

194. 25. $$\frac{\sqrt{a+x}}{\sqrt{x}}+\frac{\sqrt{a-x}}{\sqrt{x}}=\frac{\sqrt{x}}{\sqrt{b}} \qquad (1)$$

$$\frac{a+x}{x}+\frac{2\sqrt{a^2-x^2}}{x}+\frac{a-x}{x}=\frac{x}{b} \qquad (2)=(1)^2$$

$$\frac{2\sqrt{a^2-x^2}}{x}=\frac{x}{b}-\frac{2a}{x} \qquad (3)=(2) \text{ reduced.}$$

$$\frac{4a^2-4x^2}{x^2}=\frac{x^2}{b^2}-\frac{4a}{b}+\frac{4a^2}{x^2} \qquad (4)=(3)^2$$

$$x^2=4ab-4b^2 \qquad (5)=(4) \text{ reduced.}$$

Hence $$x=\pm 2\sqrt{ab-b^2}.$$

26. $$\frac{a+x+\sqrt{2ax+x^2}}{a+x}=b \qquad (1)$$

$$\frac{\sqrt{2ax+x^2}}{a+x}=b-1 \qquad (2)=(1) \text{ modified.}$$

$$\frac{2ax+x^2}{a^2+2ax+x^2}=b^2-2b+1 \qquad (3)=(2)^2$$

$$1-\frac{a^2}{(a+x)^2}=b^2-2b+1 \qquad (4)=(3) \text{ modified.}$$

$$\frac{a}{a-x}=\pm\sqrt{2b-b^2} \qquad (5)=\sqrt{(4)}$$

Hence, $$x=\pm\frac{a(1\pm\sqrt{2b-b^2})}{\sqrt{2b-b^2}}$$

194. 27. $$\frac{5-\sqrt{25-x^2}}{5+\sqrt{25-x^2}}=\frac{1}{4} \quad (1)$$

$$5-\sqrt{25-x^2}=\pm\frac{x}{2} \quad (2)$$ (*Vide* Algebra, 164.)

$$\sqrt{25-x^2}=5\mp\frac{x}{2} \quad (3)=(2) \text{ modified.}$$

$$25-x^2=25\mp 5x+\frac{x^2}{4} \quad (4)=(3)^2$$

$$x=\pm 4.$$

28 $$\frac{\sqrt{8+x}}{\sqrt{x}}+\frac{\sqrt{8-x}}{\sqrt{x}}=\frac{\sqrt{x}}{2} \quad (1)$$

$$\frac{8+x}{x}+\frac{2\sqrt{64-x^2}}{x}+\frac{8-x}{x}=\frac{x}{4} \quad (2)=(1)^2$$

$$\frac{\sqrt{64-x^2}}{x}=\frac{x}{8}-\frac{8}{x} \quad (3)=(2) \text{ reduced.}$$

$$\frac{64-x^2}{x^2}=\frac{x^2}{64}-2+\frac{64}{x^2} \quad (4)=(3)^2$$

Hence, $x=\pm 8.$

REMARK.—The answer of example 25 is $\pm 2\sqrt{ab-b^2}$, in which, if $a=8$ and $b=4$, we have $x=\pm 8$, the answer of example 28.

194. 29. $$\frac{\sqrt{20}-x}{\sqrt{20}+x}+\sqrt{5}=\frac{\sqrt{20}+x}{\sqrt{20}-x} \quad (1)$$ [of fractions.

$20-4x\sqrt{5}+x^2+20\sqrt{5}-x^2\sqrt{5}=20+4x\sqrt{5}+x^2$ (2)=(1) cleared

$$x^2\sqrt{5}+8x\sqrt{5}=20\sqrt{5} \quad (3)=(2) \text{ reduced.}$$

$$x^2+8x=20 \quad (4)=(3)\div\sqrt{5}.$$

Hence, $x=2$ or -10.

30. $$\frac{\sqrt{4x+20}}{4+\sqrt{x}}=\frac{4-\sqrt{x}}{\sqrt{x}} \quad (1)$$ [fractions.

$$\sqrt{4x^2+20x}=16-x \quad (2)=(1) \text{ cleared of}$$

$$4x^2+20x=256-32x+x^2 \quad (3)=(2)^2$$

Hence, $x=4$ or $-\frac{64}{3}$.

194. 31. $$\frac{\sqrt{x+1}}{\sqrt{x-1}}+\frac{\sqrt{x-1}}{\sqrt{x+1}}=a \quad (1)$$

[fractions.

$$x+1+x-1=a\sqrt{x^2-1} \quad (2)=(1) \text{ cleared of}$$

$$2x=a\sqrt{x^2-1} \quad (3)=(2) \text{ reduced.}$$

$$4x^2=a^2x^2-a^2 \quad (4)=(3)^2$$

Hence, $x=\pm\frac{a}{\sqrt{a^2-4}}$

32. $$\sqrt{a^2+ax}=a-\sqrt{a^2-ax} \quad (1)$$

$$a^2+ax=a^2-2a\sqrt{a^2-ax}+a^2-ax \quad (2)=(1)^2$$

$$2\sqrt{a^2-ax}=a-2x \quad (3)=(2) \text{ reduced.}$$

$$4a^2-4ax=a^2-4ax+4x^2 \quad (4)=(3)^2$$

Hence, $x=\pm\frac{a}{2}\sqrt{3}$.

33. $$\sqrt{a}+\sqrt{x}=\sqrt{ax} \quad (1)$$

$$\sqrt{a}+\sqrt{x}=\sqrt{a}\times\sqrt{x} \quad (2)=(1)$$

$$(\sqrt{a}-1)\times\sqrt{x}=\sqrt{a} \quad (3)=(2) \text{ transposed}$$

[and factored.

Hence, $x=\frac{a}{(\sqrt{a}-1)^2}$.

PROBLEMS.

195. 3. Let $25+x=$ one part, and $25-x=$ the other.

Then, $625-x^2=621$.

Hence, $x=\pm 2$;

And $25+2=27$, one part; and $25-2=23$, the other

4. Let $x+3=$ the greater number, and $x-3=$ the less.

Then, $x^2-9=216$.

Hence, $x=15$;

And $x+3=18$, the greater; $x-3=12$, the less.

5. Let $x=$ the cost of the watch.

Then, $x+\frac{x^2}{100}=75$.

Hence, $x=\$50$ *Ans.*

6. Let $x=$ the cost of the watch.

Then, $x-\frac{x^2}{100}=24$.

Hence, $x=\$40$, or $\$60$.

7. Let $x=$ the number.

Then, $x^2+4x-21=119$.

Hence $x=10$, or $x=-14$.

8. Let $x-3=$ the barrels purchased.

Then, $\frac{72}{x-3}=\frac{72}{x+3}+1$ (1) [of fractions.

$72x+216=72x-216+x^2-9$ (2)=(1) cleared

Hence, $x^2=441$, and $x=21$.

Therefore, $x-3=18$ barrels purchased at $\frac{72}{18}=\$4$ per barrel.

195. 9. Let $6+x=$ the number.

Now, $12-(6+x)=6-x$.

Then, $36-x^2=35$.

Whence, $x=\pm 1$.

Therefore, the number is 7 or 5.

10. Let $x=$ the number.

Then, $\frac{x}{10}+\frac{10}{x}=\frac{10}{3}$.

Hence, $x=30$, or $x=3\frac{1}{3}$.

11. Let $x+1=$ the rate per hour.

Then, $\frac{105}{x+1}=\frac{105}{x-1}-6$ (1)

$105x-105=105x+105-6x^2+6$ (2)=(1) cleared.

Hence, $x=6$, and $x+1=7$.

12. Let $20+x=$ greater part, $20-x=$ less.

Then, $400+40x+x^2+400-40x+x^2=1000$.

Hence, $x=10$;

$20+x=30$, greater part; $20-x=10$, less part.

13. Let $x+5=$ larger field, $x-5=$ smaller.

Then, $\frac{2800}{x+5}=\frac{2800}{x-5}-5$

Hence, $x=75$;

$x+5=80$, larger field; $x-5=70$, smaller field.

14. Let $x=$ greater, and $\frac{120}{x}=$ less.

Then, $\left(\frac{120}{x}+2\right)(x-3)=120$.

Whence, $x^2-3x=180$;

$x=15$, and $\frac{120}{x}=8$.

195. 15. Let x= A's distance, $20+x$= B's distance.

Then, $\frac{20+x}{20}$= A's rate, and $\frac{x}{15}$= B's rate.

By the question, (*Vide* Algebra, 170, Ex. 6,)

$$\frac{x^2}{15}=\frac{(20+x)^2}{20} \qquad (1)$$

$$\frac{x^2}{3}=\frac{(20+x)^2}{4} \qquad (2)=(1)\times 5$$

$$\frac{x}{\sqrt{3}}=\frac{20+x}{2} \qquad (3)=\sqrt{(2)}$$

Hence, $x=\frac{20\sqrt{3}}{2-\sqrt{3}}=40\sqrt{3}+60=129.282$ miles= A's distance.

$20+x=149.282$ miles= B's distance.

$\frac{20+x}{60}=7.464$, A's rate; and $\frac{x}{15}=8.61\frac{3}{15}$, B's rate.

196. 16. Let x= number of yards sold by 1st merchant.

Then, $x+3$= number of yards sold by 2d merchant.

$\frac{24}{x+3}$= price per yard paid by 1st merchant.

$\frac{12\frac{1}{2}}{x}$= price per yard paid by 2d merchant.

Hence, $\frac{24x}{x+3}+\frac{12\frac{1}{2}(x+3)}{x}=35$.

Whence, $x=15$ or 5, and $x+3=18$ or 8;

$\frac{24}{x+3}=1\frac{1}{3}$ or 3, and $\frac{12\frac{1}{2}}{x}=\frac{5}{6}$ or $2\frac{1}{2}$.

17. Let x=1st part, and $10-x$=2d part.

Then, $x^2=100-10x$.

Hence, $x=5(-1+\sqrt{5})$, and $10-x=5(3-\sqrt{5})$.

196. 18. Let x = 1st part, and $a-x$ = 2d.

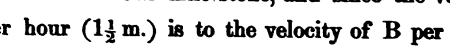
Then, $ax = a^2 - 2ax + x^2$.

Hence, $x = \frac{a}{2}(3 \pm \sqrt{5})$, and $a-x = \frac{a}{2}(-1 \mp \sqrt{5})$.

19. Let $3x$ = velocity of each per mile.

Since A overtook the geese at the 50th mile-stone, and since B overtook them at the 45th mile-stone, and since the velocity of the geese per hour ($1\frac{1}{2}$ m.) is to the velocity of B per hour ($3x$) as the distance traveled by the geese (5 miles) is to the whole distance traveled by B *in the same time*, we have

$1\frac{1}{2} : 3x :: 5 : 10x$ = distance traveled by B in overtaking the geese. Hence, $10x-5$ = the distance between A and B.



Let the line B A represent the position of the parties at the time A met the coach. Inasmuch as A had been traveling 2 hours since he overtook the geese, he must be $6x-3$ miles from the geese when he met the coach, and B must therefore be $10x-(6x-3) = 4x-2$ miles from the geese at the same moment. Now, B must overtake the geese in as many hours as the number of miles he *gains* per hour is contained in the number of miles between him and the geese; that is, in $\frac{4x-2}{3x-1\frac{1}{2}} = \frac{4}{3}$ hours.

Let the line B A represent the position of the parties when B overtook the geese. Since A travels $3x$ miles per hour, in $\frac{4}{3}$ hours he will have traveled $4x$ miles, and the coach will have gone 3 miles in the same time. Hence, the coach will be $4x+3$ miles from A, and $10x-5-(4x+3) = 6x-8$ miles from B.

If we now divide $6x-8$ into two parts, in the *ratio* of the velocities

of B. ($3x$) and the coach ($2\frac{1}{4}$), we shall know the point of meeting; that is (*vide* Arithmetic, 263),

$$3x+2\tfrac{1}{4} : 3x :: 6x-8 : \frac{24x^2-32x}{4x+3} = \text{the distance from}$$

the 45th mile-stone at which B meets the coach.

Now, by the question, he meets the coach two thirds of an hour—40 minutes—before reaching the 31st mile-stone; that is, at a point $2x+31$ miles from Washington, or, what is the same thing, $14-2x$ miles from the 45th mile-stone. Hence,

$$\frac{24x^2-32x}{4x+3}=14-2x.$$

Whence, $x=3$, and $10x-5=25$ miles *Ans.*

TWO UNKNOWN QUANTITIES.

213. 5. Let $x=$ the left, and $y=$ the right digit.

Then, $x+y=10$ (1)

And $xy+40=10y+x$ (2)

Hence, $x=4$; $y=6$; $10x+y=46$.

6. Let $x=$ one number, $y=$ the other.

Then, $x+y=7\frac{1}{2}$ (1)

And $x^3+y^3=343\frac{1}{8}$ (2)

Hence, $x=7$; $y=\frac{1}{2}$.

7. Let $x=$ one number, $y=$ the other.

Then, $x+y=47$ (1)

And $xy=546$ (2)

$x^2+2xy+y^2=2209$ (3)$=$(1)^2

$x^2+y^2=1117$ *Ans.* (4)$=$(3)-2(2)

213. 8. Let x= one, and y= the other.

Then, $x+y=20$ (1)

And $xy=99$ (2)

$x^3+3x^2y+3xy^2+y^3=8000$ (3)=(1)^3

$3xy=297$ (4)=3(2)

$3x^2y+3xy^2=5940$ (5)=(1)$\times$(4)

$x^3+y^3=2060$ *Ans.* (6)=(3)$-$(5)

9. Let x= one, and y= the other.

Then, $x+y=8$ (1)

And $xy=15$ (2)

$x^4+4x^3y+6x^2y^2+4xy^3+y^4=4096$ (3)=(1)^4

$x^2+2xy+y^2=64$ (4)=(1)^2

$2x^2+3xy+2y^2=113$ (5)=2(4)$-$(2)

$4x^3y\times 6x^2y^2+4xy^3=3390$ (6)=(5)$\times$2(2)

$x^4+y^4=706$ *Ans.* (7)=(3)$-$(6)

10. Let x= one, and y= the other number.

Then, $x+y=x^2-y^2$ (1)

And $x+y=xy$ (2)

$x-y=1$ (3)=(1)$\div(x+y)$

$x=y+1$ (4)=(3) transposed.

$y+y+1=y^2+y$ (5)=(2) combined with (4)

Hence, $y=\frac{1}{2}(1\pm\sqrt{5})$, and $x=\frac{1}{2}(3\pm\sqrt{5})$.

11 Let x= one, and y= the other number.

Then, $x^2+y^2=x^4-y^4$ (1)

$x^2+x^2=x^2y^2$ (2)

$x^2-y^2=1$ (3)=(1)$\div(x^2+y^2)$

$x^2=y^2+1$ (4)=(3) transposed.

$y^2+y^2+1=y^4+y^2$ (5)=(2) combined with (4)

Hence, $y=1.611803$, and $x=1.27203$.

213. 12. Let x= one, and y= the other number.

Then, $x^4+y^4=a$ (1)

And $xy=b$ (2)

$x^4y^4=b^4$ (3)=(2)^2

$x^4=a-y^4$ (4)=(1)

$ay^4-y^8=b^4$ (5)=(3) combined with (4)

Whence, $y^4=\frac{1}{2}(a\pm\sqrt{a^2-4b^2})$, and $x^4=\frac{1}{2}(a\mp\sqrt{a^2-4b^2})$.

214. 13. Let x= one part, and y= the other.

Then, $x+y=60$ (1)

And $2(x^2-y^2)=3xy$ (2)

$40(x-y)=xy$ (3)=(2)$\div$(1)

$x^2+2xy+y^2=3600$ (4)=(1)^2

$(x-y)^2+4xy=3600$ (5)=(4) modified.

$(x-y)^2+160(x-y)=3600$ (6)=(5) combined with (3)

$x-y=20$

Whence, $x=40$, and $y=20$.

14. Let x= one, and y= the other.

Then, $xy=77$ (1)

And $2(x^2-y^2=9(x-y)^2$ (2)

$2(x+y)=9(x-y)$ (3)=(2)$\div(x-y)$

$7x=11y$ (4)=(3) reduced.

Whence, $x=11$, and $y=7$.

15 Let x= one, and y= the other.

Then, $xy=48$ (1)

And $x^3-y^3=37(x-y)^3$ (2)

$x^2+xy+y^2=37(x-y)^2$ (3)=(2)$\div(x-y)$

$(x-y)^2+3xy=37(x-y)^2$ (4)=(3) modified.

$36(x-y)^2=144$, since $xy=48$.

Whence, $x=8$; $y=6$.

214. 16. Let x= one, and y= the other.

Then, $x^3+x^2y+xy^2+y^3=2336$ (1)

And $x^3-x^2y-xy^2+y^3=576$ (2)

$x^3+y^3=1456$ (3)=$\frac{1}{2}$ ((1)+(2))

$x^2y+xy^2=880$ (4)=$\frac{1}{2}$ ((2)−(1))

$(x+y^3-3x^2y+3xy^2)=1456$ (5)=(3) modified.

$(x+y)^3=4096$ (6)=(5) combined with (4)

Whence, $x+y=16$; and $x=11$; $y=5$.

17. Let x= one, and y= the other.

Then, $xy=320$ (1)

And $x^3-y^3=61(x-y)^3$ (2)

$x^2+xy+y^2=61(x-y)^2$ (3)=(2)÷$(x-y)$

$(x-y)^2+3xy=61(x-y)^2$ (4)=(3) modified.

$60(x-y)^2=960$

Whence, $x-y=4$; $x=20$; $y=16$.

19. Let x= one, and y= the other.

Then, $x+y=a$ (1)

And $\frac{1}{x}+\frac{1}{y}=b$ (2)

$x+y=bxy$ (3)=(2) modified.

$xy=\frac{a}{b}$ (4)

Whence, $x=\frac{a}{2}+\sqrt{\frac{a^2}{4}-\frac{a}{b}}$; $y=\frac{a}{2}-\sqrt{\frac{a^2}{4}-\frac{a}{b}}$.

20. Let x= one, and y= the other.

Then, $x^2+y^2=a$ (1)

$\frac{1}{x}+\frac{1}{y}=b$ (2)

$x+y=bxy$ (3)=(2) modified

$$x^2+2xy+y^2=b^2x^2y^2 \qquad (4)=(3)^2$$

$$2xy=b^2x^2y^2-a \qquad (5)=(4)-(1)$$

Whence, $xy=\frac{1}{b^2}(1\pm\sqrt{1+ab^2})$, and $x+y=\frac{1}{b}(1\pm\sqrt{1+ab^2})$.

215. 21. Let $2x$ = quantity of Teneriffe.

Then, x = price of Madeira per gallon.

And $x-4$ = price of Teneriffe per gallon.

$54x$ = cost of Madeira.

$2x(x-4)$ = cost of Teneriffe.

$10(54+2x)$ = price obtained for the mixture.

Hence, $54x+2x^2-8x=20x+540+576$.

Whence, $x=18$; $2x=36$.

22. Let x = side of one garden.

And $x+5$ = side of the other.

Then, $x^2+x^2+10x+25=1025$.

Whence, $x=20$; $x+5=25$.

22. Let $x-2\frac{1}{2}$ = side of one garden.

And $x+2\frac{1}{2}$ = side of the other.

Then, $x^2-5x+6\frac{1}{4}+x^2+5x+6\frac{1}{4}=1025$.

Whence, $x=22\frac{1}{2}$; $x-2\frac{1}{2}=20$; $x+2\frac{1}{2}=25$.

23. Let x = breadth, and y = the length.

Then, $4x=3y$ (1)

And $xy=384$ (2)

$x=\frac{3y}{4}$ (3) = (1) modified.

$\frac{3y^2}{4}=384$ (4) = (2) combined with (3)

$y^2=2\times256$

Whence, $y=16\sqrt{2}$; $x=12\sqrt{2}$.

215. 24. Let $4x$= length, and $3x$= breadth.

Then the whole length of the walk, including the corners, is $14x+24$. Hence,

$$(14x+24)\times 6=x^2$$

Whence, $x=85.68$; $4x=342.72$; $3x=257.04$.

25. Let x= price of mace per pound.

And y= price of cloves per pound.

Then, $80x+100y=1300$ (1)

And $$\frac{400}{y}=\frac{200}{x}+60 \qquad (2)$$

Whence, $x=10$; $y=5$.

26. Let x= one, and y= the other.

Then, $xy+y^2=84$ (1)

And $x^2-xy=16$ (2)

$4xy+4y^2=336$ (3)$=(1)\times 4$

$21x^2-21xy^2=336$ (4)$=(2)\times 21$

$21x^2-4y^2=25xy$ (5)$=$(3) and (4) combined

If $x=my$, then, $21m^2-25m=4$; whence, $m=\frac{4}{3}$ or $-\frac{1}{7}$.

Then, $x=\frac{4y}{3}$, or $x=-\frac{y}{7}$.

Substitute $\frac{4y}{3}$ for x in (1), and we have $x=\pm 8$, $y=\pm 6$.

Substitute $-\frac{y}{7}$ for x in (1), and we have $x=\mp\sqrt{2}$, $y=\pm 7\sqrt{2}$.

27. Let x= one, and y= the other.

Then, $x^2+y^2=\pm 13$ (1)

And $xy=\pm 6$ (2)

$x^2+2xy+y^2=\pm 25$ (3)$=(1)+(2)\times 2$

$x^2-2xy+y^2=\pm 1$ (4)$=(1)-(2)\times 2$

$$x+y=\pm 5 \text{ or } \pm 5\sqrt{-1}$$
$$x-y=\pm 1 \text{ or } \pm \sqrt{-1}$$

Whence, $x=\pm 3$; $y=\pm 2$; or, $x=\pm 3\sqrt{-1}$; $y=\pm 2\sqrt{-1}$.

215. 38. Let $x=$ one, $y=$ the other.

Then, $x^6y^2-x^2y^6=147600$ (1)

And $x^5y+xy^5=820$ (2)

$x^5y-xy^5=180$ (3)$=$(1)$\div$(2)

$x^3y=500$ (4)$=\frac{1}{2}$((2)$+$(3))

$xy^3=320$ (5)$=\frac{1}{2}$((2)$-$(3))

$x^4y^4=160000$ (6)$=$(4)$\times$(5)

$x^2y^2=\pm 400$ (7)$=\sqrt{(6)}$

$xy=\pm 20 \text{ or } \pm 20\sqrt{-1}$ (8)

$x^2=\pm 25 \text{ or } \mp 25\sqrt{-1}$ (9)$=$(4)$\div$(8)

$x=\pm 5 \text{ or } \pm 5\sqrt{-1}$, or $x=\pm\sqrt{\mp 25\sqrt{-1}}$;

$y^2=\pm 16 \text{ or } \mp 16\sqrt{-1}$ (10)$=$(5)$\div$(8)

$y=\pm 4 \text{ or } \pm 4\sqrt{-1}$, or $y=\pm\sqrt{\mp 16\sqrt{-1}}$.

43. Let $x=$ the number.

Then, $\frac{x+4}{x}=\frac{3x}{x-4}$ (1)

$x^2-16=3x^2$ (2)$=$(1) cleared.

Whence, $x=\pm 2\sqrt{-2}$.

ARITHMETICAL PROGRESSION.

233. 16. Let x= the time.

The distance traveled by A will be x^2. (*Vide* 228, Ex. 1.)

The distance traveled by B will be $9(x-2)$.

Hence, $x^2=9x-18$; whence, $x=3$ or 6.

GEOMETRICAL PROGRESSION.

238 4. Let x, xy, xy^2 represent the series.

Then, $x+xy^2=52$ (1)

And $x^2y^2=100$ (2)

$xy=10$ (3)$=\sqrt{(2)}$

$x+10y=52$ (4)=(1) with 10 in place of xy

Whence, $x=2$, $y=5$, and the series is 2, 10, 50.

7. Let x, xy, xy^2, xy^3 represent the series.

Then, $xy^3-xy=24$ (1)

And $x+xy^3 : xy+xy^2 :: 7 : 3$ (2)

$1+y^3 : y+y^2 :: 7 : 3$ (3)=(2) couplet $\div x$.

$1-y+y^2 : y :: 7 : 3$ (3)=(3) couplet $\div(1+y)$.

$1+y^2 : y :: 10 : 3$ (5) *vide* IV.

Whence, $3y^2-10y=-3$; $y=3$; $x=1$.

QUESTIONS FOR EXAMINATION.

279. 1. $\frac{x^{3n}-1}{x^n-1}+\frac{1-x^{2n}}{1+x^n}=x^{2n}+x^n+1+1-x^n=x^{2n}+2.$

The answers are then $1+2=3$; $4+2=6$; $9+2=11$; $16+2=18$, etc.

2. $a^{-1}\left(1+\frac{x}{a}\right)^{-1}=\frac{1}{a+x}$. The answers are then

$\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{8}$; 1; $1\frac{1}{2}$; 2; $2\frac{1}{2}$, etc.

3. $a^{\frac{4}{3}}\left(1-\frac{x^2}{a^2}\right)^{\frac{2}{3}}=(a^2-x^2)^{\frac{2}{3}}$. The answers are then

2; 4; 9.

4. $a^{-1}\left(1+\frac{x^4}{a^4}\right)^{\frac{1}{4}}=\frac{1}{a^2}(a^4+x^4)^{\frac{1}{4}}$. The answers are $\frac{1}{2}$ and $\frac{3}{4}$.

5. *Ans.* $2+\frac{9x-35}{x^2-7x+12}$.

6. *Ans.* $\frac{7x-29}{x^2-7x+12}$.

7. *Ans.* $1+\frac{9}{x-4}$.

8. *Ans.* $1+\frac{7x-29}{x^2-6x+9}$.

9. *Ans.* $x=72$.

10. *Ans.* $x=9$.

11. *Ans.* $x=\frac{(b-a)d}{c}$.

12. *Ans.* $x=\frac{b^4+a^2+ab+b^2-a^4}{(a^2-b^2)^2}$.

282. 38. Let $7x=$ the number.

Then, $7x+7-x=73$.

Whence, $7x=77$ *Ans.*

39. Let $x-y$, x, $x+y$ represent the numbers.

Then, $9(x-y)=5(x+y)$ (1)

And $3x=63$ (2)

Whence, $x=21$; $y=6$. The numbers are 15, 21, 27.

282. 40. Let $20x$ = the distance.

Then, x^2 = the distance which B traveled.

And $27+8x$ = the distance which A traveled.

Hence, $x^2+8x+27=20x$.

Whence, $x=9$ or 3, and $20x=180$ or 60.

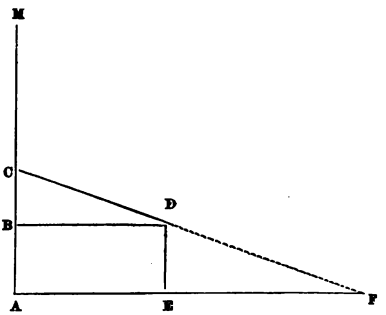
41. Let x = the time.

Then the distance traveled by A is $\frac{125x-5x^2}{2}$. (*Vide* 227,

And the distance traveled by B is $\frac{75x+5x^2}{2}$. (1), (2), (3).)

Whence, $\frac{125x-5x^2}{2}+\frac{75x+5x^2}{2}=500$; $x=5$.

282. 42. Let A M represent the tree previous to breaking, and let A F represent the width of the river, and C D that part of the tree which breaks off. We have A M = 100; A F = 200; D E = 20 = A B.



Let B C = x; then D C = $80-x$, and B D = $4\sqrt{400-10x}$.

Now, B C : B D :: D E : E F; that is, $x : 4\sqrt{400-10x} :: 20 :$ E F.

Hence, $\quad$ E F = $\frac{80}{x}\sqrt{400-10x}$.

Now, $\quad$ B D + E F = A F;

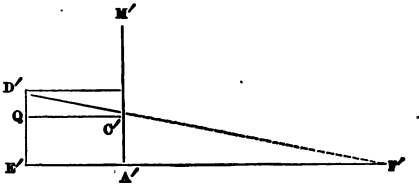
Or, $\quad 4\sqrt{400-10x}+\frac{80}{x}\sqrt{400-10x}=200$.

Whence, $\quad x^3+250x^2-1200x=16000$;

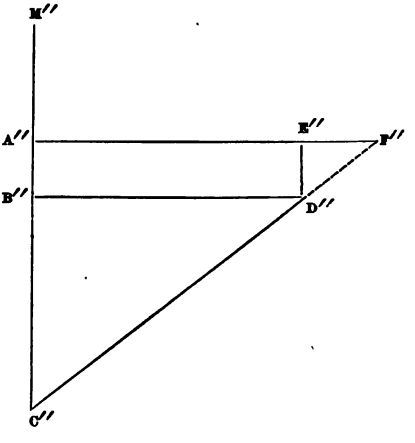
And $\quad x=10.472$, or $x=-6.003$, or $x=-254.469$.

Hence, the tree broke 30.472 feet above A, or 13.997 above A; or—see below.—234.469 feet below the plane A″ F″.

The second root, —6.003, shows that the tree might have fallen the *other way* and have fulfilled the other conditions of the problem.



The third root, —254.469, shows that a line M″ C″ extending perpendicularly through the plane A″ F″ may be broken *below* the plane, so as to fulfill the other conditions of the question.



THE END.